

Pratybes 1

Paavyzdys 1.

$$f(x, y) = x + y \quad (\text{lygiu linija } x + y = c).$$

$$g(x, y) = x^2 + y^2 = 1 \quad (\text{apskritimas, } k=1)$$

$$y = -x + c.$$

(paveikslėlyje graf!

$f(x, y)$ max ir min taškus galite

paaukštinti ir grafiko. Bet naudokite
atlikti ir "techninį" darbą - kreivės:

1 būdas, Parametrizavime $g(x, y)$:

$$\boxed{x = \cos \theta, \quad y = \sin \theta.}$$

$0 \leq \theta \leq 2\pi$
vidan
~~vidan~~

gauname vieną parametrą

$$\tilde{f}(\theta) = \cos \theta + \sin \theta$$

testame stationary, fably

$$\tilde{f}'(\theta) = -\sin \theta + \cos \theta$$

$$\tilde{f}'(\theta) = 0$$

$$\tan \theta = 1$$

$$\theta = \frac{\pi}{4} + \pi n$$

$n = 0, 1$

$$\tilde{f}''(\theta) = -(\cos \theta + \sin \theta) = -\tilde{f}(\theta)$$

$n=0$ $x^0 = \frac{\sqrt{2}}{2}, y^0 = \frac{\sqrt{2}}{2}$ $f(x^0, y^0) = \sqrt{2}$

$$\tilde{f}''(\theta) = -\sqrt{2} < 0 \quad (\text{makes local})$$

$n=1$ $x = -\frac{\sqrt{2}}{2}, y = -\frac{\sqrt{2}}{2}$ $f(x, y) = -\sqrt{2}$

$$\tilde{f}''(\theta_1) = \sqrt{2} \quad (\text{minimum local})$$

$$L(x, y, \lambda) = x + y + \lambda (x^2 + y^2 - 1)$$

Gaunama lygčių sistema:

$$\begin{cases} 1 + 2\lambda x = 0 \\ 1 + 2\lambda y = 0 \\ x^2 + y^2 - 1 = 0. \end{cases}$$

$$x = y = -\frac{1}{2\lambda}$$

$$\lambda \neq 0.$$

$$\frac{1}{2\lambda^2} = 1 \Rightarrow \lambda^2 = \frac{1}{2}$$

$$\lambda = \pm \frac{\sqrt{2}}{2}$$

$$(x^0, y^0) = -\frac{\sqrt{2}}{2}$$

$$(x^1, y^1) = \frac{\sqrt{2}}{2}$$

Gauname antrojo laipsnio sprendimus

Proz. 2

$$f(x, y) = (x+y)^2, \quad g(x, y) = x^2 + y^2 - 1$$

$$(x^0, y^0) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \quad (x^1, y^1) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

Aku tashar yon maksimumus tash

$$(x^2, y^2) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \quad (x^3, y^3) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$f(x^2, y^2) = f(x^3, y^3) = 0 \quad \begin{matrix} \text{minimum} \\ \text{tash} \end{matrix}$$

Bet liggis luyis nara ~~*~~ tangentini (jis
vertinlar neshtampal) ni aprobojimus
lietuvimiss.

Tadbu o'ra $\lambda = 0$ ni faxi o'q.

bilmei shuluku qantu liggis sistem.
(bu tashar yon neshtampal minimumus
tashar).

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$$f(x, y) = x^2 y$$

$$g(x, y) = x^2 + y^2 - 3$$

$$\begin{cases} 2xy + 2\lambda x = 0 & (*) & x(y + \lambda) = 0 \\ x^2 + 2\lambda y = 0 & (**) \\ x^2 + y^2 - 3 = 0 & (***) \end{cases}$$

15 (*) $x = 0$ arba $y = -\lambda$.

$$x = 0 \Rightarrow y = \pm \sqrt{3} \Rightarrow \lambda = 0$$

$$y = -\lambda \Rightarrow x^2 = 2y^2 \Rightarrow y = \pm 1$$

Gauname 6 kritinių taškų

15 analizuojame juos.