Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

$\mathbb{B}^{-1}$-CONVEX FUNCTIONS AND SOME IMPORTANT PROPERTIES

GABIL ADILOV[1] and ILKNUR YESILCE[2]

[1] *Department of Mathematics, Faculty of Education, Akdeniz University*

07058 Antalya, Turkey

E-mail: gabiladilov@gmail.com

[2] *Department of Mathematics, Faculty of Science and Letters, Mersin University*

33343 Mersin, Turkey

E-mail: ilknuryesilce@gmail.com

A subset U of $\mathbb{R}^n_{++}$ is $\mathbb{B}^{-1}$-convex if for all $x, y \in U$ and all $\lambda \in [1, \infty)$ one has

$$\lambda x \wedge y = (\min\{\lambda x_1, y_1\}, \min\{\lambda x_2, y_2\}, ..., \min\{\lambda x_n, y_n\}) \in U.$$

This definition is given in [1]. There are many works related to $\mathbb{B}^{-1}$-convex sets [1],[2]. Furthermore, this kind of sets have applications in economics [3].

For $U \subset \mathbb{R}^n$, a function $f : U \to \mathbb{R} \bigcup \{\pm\infty\}$ is called $\mathbb{B}^{-1}$-convex function if $epi f = \{(x, \mu) : x \in U, \mu \in \mathbb{R}, f(x) \leq \mu\}$ is $\mathbb{B}^{-1}$-convex set.

A necessary and sufficent condition for $\mathbb{B}^{-1}$-convex functions is given as follows:

THEOREM 1. *Let $U \subset \mathbb{R}^n_{++}$ and $f : U \to \mathbb{R}_{++}$. The function f is $\mathbb{B}^{-1}$-convex if and only if the set U is $\mathbb{B}^{-1}$-convex and one has inequality:*

$$f(\lambda x \wedge y) \leq \lambda f(x) \wedge f(y)$$

for all $x, y \in U$ and all $\lambda \in [1, \infty)$.

In this work, important properties concerned with this concept are proved.

REFERENCES

[1] G. Adilov and I. Yesilce. $B^{-1}-$convex Sets and $B^{-1}-$measurable Maps. *Numerical Functional Analysis and Optimization*, **33** (2):1–11, 2012.

[2] G. Tinaztepe, I. Yesilce and G. Adilov. Separation of $B^{-1}-$convex Sets by $B^{-1}-$measurable Maps. *Journal of Convex Analysis*, **21** (2): 2014.

[3] W. Briec, Q. B. Liang. On Some Semilattice Structures for Production Technologies. *European Journal of Operational Research*, **215** 740-749, 2011.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

STABLE SCHEMES FOR SOME PARABOLIC EQUATIONS

NADEZHDA AFANASYEVA * and PETR VABISHCHEVICH **

* *North-Eastern Federal University*

Belinskogo 58, 677000, Yakutsk, Russia

E-mail: `afanasieva.nm@gmail.com`

** *Nuclear Safety Institute, Russian Academy of Sciences*

B. Tulskaya 52, 115191 Moscow, Russia

E-mail: `vabishchevich@gmail.com`

Boundary value problems for second order parabolic equations are basic problems in applied mathematical modeling. We consider problems for second order parabolic equations with a non-selfadjoint operator, which is a weighted sum of selfadjoint elliptic operators. For example, such problems are typical in calculation of pressure for modeling of multiphase fluid filtration [1; 2], when the compressibility of the each phase has a major influence.

We consider the problem of constructing unconditionally stable schemes. The study is based on the record of an original equation in the form of convection-diffusion equation. When compressible medium is considered, the convective operator of the problem is non skew-symmetric form and the norm of the exact solution can increase with time. We construct unconditionally ρ-stable two level schemes by considering non-selfadjointness of problem operator. Capabilities of using the explicit-implicit time approximations [3] and approximations based on the introduction of a new unknown variable [4] are noted. Stability conditions are formulated on the basis of the general theory of stability (correctness) for operator-difference schemes, which was developed by A.Samarskii [5].

For decreasing computational costs, we propose the splitting schemes (splitting by individual phases). The transition to a new time level is associated with solution of a number of simpler problems with selfadjoint operators.

REFERENCES

[1] Peaceman D. W. *Fundamentals of Numerical Reservoir Simulation.* Amsterdam: Elsevier Scientific Pub. Co, 2000.

[2] Aziz K., Settari A. *Petroleum Reservoir Simulation.* London: Applied Science Publishers, 1979.

[3] Vabishchevich P.N., Vasil'eva M.V. Explicit-implicit schemes for convection-diffusion-reaction problems. *Numerical Analysis and Applications*, **5** (4):297–306,2012.

[4] Afanas'eva N.M, Vabishchevich P.N., Vasil'eva M.V. Unconditionally stable schemes for convection-diffusion problems. *Russian Mathematics*, **57** (3):1–11, 2013.

[5] Samarskii A. A. *The Theory of difference schemes.* New York: Marcel Deccer, 2001.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

SOLVING BOUNDARY VALUE PROBLEMS FOR SECOND ORDER SINGULARLY DISTURBED DELAY DIFFERENTIAL EQUATIONS BY ε-APPROXIMATE FIXED-POINT METHOD

ZBIGNIEW BARTOSZEWSKI[1] and ANNA BARANOWSKA[2]

[1] *Gdańsk University of Technology*

Narutowicza 11/12, 80-233 Gdańsk, Poland

E-mail: zbart@mif.pg.gda.pl

[2] *The Institute of Oceanology of the Polish Academy of Sciences*

Powstańcw Warszawy 55, 81-712 Sopot, Poland

The talk will be devoted to numerical solution of boundary value problems for second order singularly perturbed delay differential equations of the form

$$\begin{aligned} \epsilon y''(x) &= f(x, y(x), y'(x), y(\alpha(x))), \qquad a \le x \le b, &(1)\\ y(x) &= \phi(x) \quad \text{for} \quad x \le a, \qquad y(b) = \psi. &(2) \end{aligned}$$

where the functions f, ϕ and α,

$$\begin{aligned} &f : D \to R, \quad D = \{(t, z_1, z_2, z_3) : a \le t \le b, -\infty \le z_i \le +\infty\},\\ &\phi : [\gamma, a] \to R, \quad \alpha : [a, b] \to (-\infty, b], \quad \gamma = \min_{a \le x \le b} \alpha(x) \end{aligned}$$

are continuous and $0 < \epsilon \ll 1$. Problem $(1) - (2)$ is reduced to a linear boundary value problem for the system of two equations of the form

$$\begin{aligned} &\mathcal{E}\frac{d\vec{y}(x)}{dx} = g(x, \vec{y}(x), \vec{y}(\alpha(x))), \quad a \le x \le b, &(3)\\ &P_1\vec{y}(x) + P_2(b - a + x)\vec{y}(b - a + x) = \Phi(x), \quad x \in [a - \tau, a], &(4) \end{aligned}$$

with suitably chosen 2×2 matrices P_1, P_2 and the diagonal matrix $\mathcal{E} = \mathrm{diag}\,\{1, \epsilon\}$. For solving problem $(3) - (4)$ we construct a numerical method based on Theorem 2.1 in [1].

Problem $(3) - (4)$ is reduced to a fixed-point with the help of an auxiliary linear problem

$$\begin{aligned} &\vec{y}'(x) = B\vec{y}(x) + \vec{v}(x), \quad x \in [a, b], &(5)\\ &P_1\vec{y}(a) + P_2(b)\vec{y}(b) = \Phi(a), &(6) \end{aligned}$$

and the fixed-point $\vec{v}$ is approximated by a cubic spline.

REFERENCES

[1] Z. Bartoszewski. A new approach to numerical solution of fixed-point problems and its application to delay differential equations. *Appl. Math. Comput.*, **215** :4320–4331, 2010.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON SHAPE PRESERVING SMOOTHING SPLINES [1]

SVETLANA ASMUSS[1] and NATALJA BUDKINA[2]

[1] *Institute of Mathematics and Computer Science, University of Latvia*

Rainis blvd 29, Riga, LV-1459, Latvia

[2] *Riga Technical University*

Meza street 1/4, Riga, LV-1048, Latvia

E-mail: `svetlana.asmuss@lu.lv, natalja.budkina@rtu.lv`

In many approximation problems it is important that solutions preserve some shape properties such as positivity, monotonicity and/or convexity. The additional shape preserving conditions are usually considered as the positivity/negativity constraints on the derivative (or the derivatives) of approximants. Taking into account that such conditions can be described by using convex sets, we start with the generalized problem on smoothing splines in a convex set

$$\|Tx\|^2 + \|R(Ax - v)\|^2 \longrightarrow \min_{Bx \in C}, \tag{1}$$

where $T:\ X\ \to\ Y$, $A : X \to \mathbb{R}^n$ and $B : X \to Z$ are linear continuous operators in Hilbert spaces X, Y and Z, $C \subset Z$ is a convex closed set, $v \in \mathbb{R}^n$ and $R = diag(\sqrt{\rho_i})_{i=1,\dots,n}$ is the diagonal matrix with nonnegative parameters $\rho_i,\ i = 1, \dots, n$.

We consider the discretization of (1) by taking appropriate operators $B_m : X \to\ \mathbb{R}^m$ and convex closed sets $C_m \subset \mathbb{R}^m$ instead of B and C respectively

$$\|Tx\|^2 + \|R(Ax - v)\|^2 \longrightarrow \min_{B_m x \in C_m}. \tag{2}$$

We investigate (2) as the general case and illustrate our approach by some specific shape preserving approximation problems in Sobolev space. Convergence results are considered. Numerical and graphical examples are developed on the basis of our previous works [1; 2].

REFERENCES

[1] S. Asmuss and N. Budkina. Splines in convex sets under constraints of two-sided inequality type in a hiperplane. *Mathematical Modelling and Analysis*, **13** (4):461–470, 2008.

[2] S. Asmuss and N. Budkina. On smoothing problems with one additional equality condition. *Mathematical Modelling and Analysis*, **14** (2):159–168, 2009.

[1]The support of the ESF project 2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045 is kindly announced.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON LIMIT CYCLES IN PERIOD ANNULI [1]

SVETLANA ATSLEGA[1]

Institute of Mathematics and Computer Science, University of Latvia

Rainis blvd. 29, Riga LV-1459, Latvia

E-mail: `svetlana.atslega@llu.lv`

We consider Hamiltonian systems of various types [2; 3]. The analysis is provided showing how Hamiltonian systems can be perturbed in order to have limit cycles in period annuli of original systems. In particular the Hamiltonian systems

$$\begin{cases} x' = y, \\ y' = -x^3(x^2-1), \end{cases} \tag{1}$$

and

$$\begin{cases} x' = -y(y^2-p_1)(y^2-p_2)(y^2-p_3), & p_1 < p_2 < p_3 \\ y' = x(x^2-q_1)(x^2-q_2)(x^2-q_3), & q_1 < q_2 < q_3 \end{cases} \tag{2}$$

are considered.

REFERENCES

[1] S. Atslega and F. Sadyrbaev. On periodic solutions of Liénard Type Equations. *Mathematical Modelling and Analysis*, **18** (5):708–716, 2013.

[2] R. Ashegi, H.R.Z. Zangeneh. Bifurcations of limit cycles for a quintic Hamiltonian system with a double cuspidal loop. *Computers and Mathematics with Applications*, **59** (4):1409–1418, 2010.

[3] T. Johnson, W. Tucker. An improved lower bound on the number of limit cycles bifurcating from a Hamiltonian planar vector field of degree 7. *Int. Journal of Bifurcation and Chaos*, **20** (5):1451–1458, 2010.

[4] R. Winkel. A transfer principle in the real plane from nonsingular algebraic curves to polynomial vector fields. *Geometriae Dedicata*, **79** :101–108, 2000.

[1] **Acknowldegement**. The support of the ESF project 2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045 is kindly announced.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

DISCRETE MODELLING OF TWO NEURONS[1]

MARUTA AVOTIŅA[1] and INESE BULA[1,2]

[1] *Faculty of Physics and Mathematics, University of Latvia*
Zeļļu iela 8, Rīga LV-1002, Latvia
[2] *Institute of Mathematics and Computer Science, University of Latvia*
Raiņa bulvāris 29, Rīga LV-1459, Latvia
E-mail: marutaavotina@inbox.lv, ibula@lanet.lv

Networks of two neurons have been used as prototypes for understanding the dynamics of large scale networks and many studies have been devoted to the qualitative analysis of such networks.

We consider a two difference equation system of the form:

$$\begin{cases} x_{n+1} = \beta x_n - g(y_n), \\ y_{n+1} = \beta y_n - g(x_n), \end{cases} \quad n = 0, 1, 2, ... \text{ with } \quad g(u) = \begin{cases} a, & u \geq \alpha, \\ 0, & -\alpha < u < \alpha, \\ -a, & u \leq -\alpha, \end{cases} \tag{1}$$

where $\beta \in]0, 1[$ is a constant and $a > 0$, $\alpha > 0$.

System (1) can be viewed as a discrete version of the two-neuron network model:

$$\left\{ \frac{dx}{dt} = -Ax + Bg(y[t]), \quad \frac{dy}{dt} = -Ax + Bg(x[t]), \qquad n = 0, 1, 2, ... \right.$$

where $[\cdot]$ denotes the greatest integer function, $\alpha > 0$ represents the internal decay rate, parameter $\beta > 0$ measures the synaptic strength, $x(t)$ and $y(t)$ denote the activation of the corresponding neurons and g is a signal function. Interest of such two-neuron model comes from [3]. Two-neuron model (1) with different signal function was investigated in [2; 4]. In [1] we studied an one-neuron model with similar signal function as (1). In this talk we present the dynamics of system (1). As well as some interesting results that have been obtained for the limiting behavior of solutions and the existence of periodic solutions of this system.

REFERENCES

[1] A. Anisimova, M. Avotina and I. Bula . Periodic Orbits of Single Neuron Models with Internal Decay Rate $0 < \beta \leq 1$. *Mathematical Modelling and Analysis*, **18,** 325–345, 2013.

[2] H. Bin, L. Huang and G. Zhang. Convergence and Periodicity of Solutions for a Class of Difference Systems . *Advances in Difference Equations*, **2006,** 1–10, 2006.http://dx.doi.org/10.1155/ADE/2006/70461

[3] J.J. Hopfield. Neurons with graded response have collective computational properties like those of two-state neurons. *Proceedings of the National Academy of Sciences of the United States of America*, **10,** 3088–3092, 1984.

[4] Z. Yuan, L. Huang and Y. Chen. Convergence and Periodicity of Solutions for a Discrete-Time Network Model of Two Neurons . *Mathematical and Computer Modelling*, **45,** 941–950, 2002.http://dx.doi.org/10.1016/S0895-7177(02)00061-4

[1] This work was partially supported by the grant Nr. 345/2012 of the Latvian Council of Science.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

REDUCED BASIS METHOD APPLIED TO AN OBSTACLE PROBLEM

EDUARD BADER

Aachen Institute for Advanced Study in Computational Engineering Science - RWTH Aachen

Schinkelstraße 2, 52062 Aachen, Germany

E-mail: `bader@aices.rwth-aachen.de`

We present fully online/offline-efficient reduced basis methods for problems in nonlinear solid mechanics, particularly for obstacle problems. Obstacle and especially contact problems, which can be formulated as variational inequalities, are difficult to treat due to two concerns: (i) the region of contact is unknown in advance. Hence, these problems are solved on convex subsets instead of the whole space. (ii) in the case the problem can formulated with Lagrange multipliers their regularity is usually weak, see [1].

The development of efficient tools for solving variational inequalities have been introduced recently in [2]. The main drawback is that this method is not fully online/offline-efficient, since the online error estimation for the reduced solution depends on the high dimension of the FE-problem.

In this talk, we present alternative, fully online/offline-efficient approaches, which transform the inherent nonlinearity of obstacle problems, such that a treatment by the Empirical Interpolation Method (EIM) is possible. We are able to incorporate, by a penalty approach, the inequality constraint of the obstacle problem to get a nonlinear variational equality. This allows us (i) to compute proper reduced solution and (ii) to evaluate online/offline-efficiently an *a posteriori* error estimator for the reduced solution. Altogether, we are able to achieve relevantly high speed-ups for the reduced problem.

REFERENCES

[1] R. Glowinski. *Numerical Methods for Nonlinear Variational Problems.* Springer, 1984.

[2] B. Haasdonk, J. Salomon and B. Wohlmuth. A reduced basis method for parametrized variational inequalities. *SIAM J. Numer. Anal.*, **50** (5):2656–2676, 2012.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

MELLIN TRANSFORMS OF DIRICHLET L-FUNCTIONS

AIDAS BALČIŪNAS

Institute of Informatics and Mathematics

Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: aidas.balciunas@mii.stud.vu.lt

Let $s=\sigma+it$ be a complex variable, and χ Dirichlet character modulo $q, q>1$. The modified Mellin transform of Dirichlet L- function is defined by the integral

$$\mathcal{Z}_1(s,\chi)=\int_1^\infty \left|L\left(\frac{1}{2}+ix,\chi\right)\right|^2 x^{-s}dx, \quad \sigma>1.$$

We consider meromorphic contiuanation of $\mathcal{Z}_1(s,\chi)$ to the whole complex plane seperatlly for principal character χ_0 and primitive character χ modulo q. Let

$$a(q)=\sum_{p|q}\frac{\log p}{p-1},$$

and $\varphi(q)$ denote the Euler totient function. Moreover let $G(\chi)$ denote the Gauss sum, B_k stand for the k th Bernoulli number, γ be the Euler constant. and

$$b=\begin{cases}0 & \text{if} \quad \chi(-1)=1,\\ 1 & \text{if} \quad \chi(-1)=-1.\end{cases}$$

THEOREM 1. *The function $\mathcal{Z}_1(s,\chi)$ has a meromorphic continuation to the whole complex plane. It has a double pole at the point $s=1$, and its Laurent expansions at this point are*

$$\mathcal{Z}_1(s,\chi_0)=\frac{\varphi(q)}{q}\left(\frac{1}{(s-1)^2}+\frac{2\gamma+2a(q)-\log 2\pi}{s-1}\right)+...,$$

$$\mathcal{Z}_1(s,\chi)=\frac{i^b}{\sqrt{q}}\sum_{a=1}^{q}\overline{\chi}(a)(q,a-1)\left(\frac{1}{(s-1)^2}+\frac{2\gamma-\log\frac{q}{(q,a-1)^2}-\log 2\pi}{s-1}\right)+...,$$

corresponding for the principal and primitive Dirichlet characters mod q.

Other singularities are the simple poles at the points $s=-(2k-1), k\in\mathbb{N}$, and

$$\operatorname*{Res}_{s=-(2k-1)}\mathcal{Z}_1(s,\chi_0)=\frac{\varphi(q)}{q}\frac{i^{-2k}(1-2^{-(2k-1)})}{2k}B_{2k},$$

$$\operatorname*{Res}_{s=-(2k-1)}\mathcal{Z}_1(s,\chi)=\frac{i^{b-2k}}{q}\sum_{a=1}^{q}\overline{\chi}(a)(q,a-1)\frac{1-2^{-(2k-1)}}{2k}B_{2k}.$$

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

EFFICIENT INTERPOLATION OVER INFINITE AND FINITE FIELDS VIA HANKEL POLYNOMIALS

IVAN BARAVY

Faculty of Applied Mathematics, St. Petersburg State University

Universitetskij pr. 35, Peterhof, 198504, St. Petersburg, Russia

E-mail: `ivan_baravy@apmath.spbu.ru`

Interpolation problem, as it was initially posed in terms of polynomials, is well researched. However, further mathematical developments extended it significantly. Trigonometric interpolation is widely used in Fourier analysis, while its generalized representation as exponential interpolation is applicable to such problem of mathematical physics as modelling of Ziegler–Biersack–Littmark repulsive interatomic potentials. Formulated for finite fields, this problem arises in decoding Reed–Solomon codes.

This paper shows the relation between different interpretations of the problem through the class of matrices of special structure:

$$A = [a_{j+k}]_{j,k=0}^{n-1}.$$

Such supersymmetric matrices are called Hankel matrices. Corresponding determinants

$$H_k(x) = \begin{vmatrix} a_0 & a_1 & \dots & a_k \\ a_1 & a_2 & \dots & a_{k+1} \\ \cdots & & & \cdots \\ a_{k-1} & a_k & \dots & a_{2k-1} \\ 1 & x & \dots & x^k \end{vmatrix}$$

are Hankel polynomials. Their properties were denoted by F. Joachimsthal [1] and later substantially researched by P. Henrici [2].

The introduced interpolation method is based on strong connection between consecutive Hankel polynomials

$$c_k^2 H_{k-2}(x) + c_{k-1}^2 H_k(x) + (c_k h_{k-1,1} - c_{k-1} h_{k1} - c_k c_{k-1} x) H_{k-1}(x) \equiv 0.$$

This method is suitable for both infinite and finite fields and still allows efficient parallel algorithms. Especially fast division-free implementation was constructed for polynomials over $GF(2)$ as a modification of Berlekamp–Massey algorithm.

This research was supported by the St. Petersburg State University research grant **9.38.674.2013** and by the research grant of the company RAIDIX[1].

REFERENCES

[1] F. Joachimsthal. Bemerkungen über den Sturmschen Satz. *Journal für die reine und angewandte Mathematik*, **48** 386–416, 1854.

[2] P. Henrici. *Applied and Computational Complex Analysis*. Wiley, New York, 1974.

[1] http://www.raidixstorage.com

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ABOUT THE NUMERICAL APPROXIMATION OF THE DOUBLE POROSITY CONSOLIDATION MODEL

N. BOAL and F.-J. GASPAR and F.-J. LISBONA and P.N. VABISHCHEVICH

Department of Applied Mathematics-Zaragoza University

María de Luna 3; 50018-Zaragoza; Spain

E-mail: nboal@unizar.es

Biot described the coupling between the fluid pressure and stress fields using linear elastic theory for a homogeneous porous medium, however fractured rock formations form important subsurface flow systems with a high degree of local heterogeneity. Aquifer or reservoir systems which contain both open fractures and interconnected pore spaces are often modelled as a double porosity medium. The poroelastic behaviour of double-porosity rocks depends on both the flow and mechanical parameters of the fracture as well as the matrix. The constitutive equations can be formulated by extending Biot's concepts of poroelasticity to double-porosity rocks. The governing equations of the double porosity consolidation problem take the form

$$-\mu\tilde{\Delta}\mathbf{u} - (\lambda+\mu)\operatorname{grad}\operatorname{div}\mathbf{u} + \beta_1\operatorname{grad} p_1 + \beta_2\operatorname{grad} p_2 = \mathbf{g}, \tag{1}$$

$$\alpha_1\frac{\partial p_1}{\partial t} + \beta_1\frac{\partial}{\partial t}(\operatorname{div}\mathbf{u}) - \frac{\kappa_1}{\eta}\Delta p_1 - \kappa(p_2 - p_1) = 0, \tag{2}$$

$$\alpha_2\frac{\partial p_2}{\partial t} + \beta_2\frac{\partial}{\partial t}(\operatorname{div}\mathbf{u}) - \frac{\kappa_2}{\eta}\Delta p_2 + \kappa(p_2 - p_1) = 0, \tag{3}$$

where $\mathbf{u}$ is the displacement vector of the solid skeleton, p_1 is the fluid pressure for pores and p_2 is the fluid pressure for fissures. The constants λ and μ are the Lamé coefficients, and the constants β_1 and β_2 measure the change of porosities due to an applied volumetric strain. The constants α_1 and α_2 measure the compressibility of pores and fissures respectively. The constants k_1 and k_2 are the permeability of the respective pores, the constant μ denotes the viscosity of the pore fluid, κ is a constant measuring the exchange of fluid between pores and fissures and $\mathbf{g}$ is a force density.

For the numerical approximation of these equations we use finite difference schemes on staggered grids. Energy stability estimates for the fully discrete equations are obtained and convergence results are given. To illustrate the theoretical results some numerical experiments are presented.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

SECOND DERIVATIVE DIAGONALLY IMPLICIT MULTISTAGE INTEGRATION METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

MICHAŁ BRAŚ

AGH University of Science and Technology, Faculty of Applied Mathematics

al. Mickiewicza 30, 30-059 Kraków, Poland

E-mail: bras@agh.edu.pl

We consider the class of second derivative general linear methods (SGLMs) for ordinary differential equations [2; 3]. These methods are an extension of general linear methods [4]. SGLMs use r quantities (external stages) as imput and output. At each step s internal stages are calculated using first and second derivative of the solution of ODE.

Second derivative diagonally implicit multistage integration methods (SDIMSIMs) form a subclass of SGLMs and they are characterized by the following properties.

1. Coefficients matrices $\mathbf{A}$ and $\overline{\mathbf{A}}$ are lower triangular with the same parameters λ and μ on the diagonal respectively.
2. Coefficients matrix $\mathbf{V}$ is a rank one matrix with nonzero eigenvalue equal to one to guarantee preconsistency.
3. The order p, the stage order q, the number of external stages r, and the number of internal stages s are all approximately equal.

In the talk we review the order conditions of SDIMSIMs and describe the construction of methods of order $p = 1, 2, 3$ and 4 with desired stability properties [1].

REFERENCES

[1] A. Abdi, M. Braś and G. Hojatti. On the construction of second derivative diagonally implicit multistage integration methods for ODEs. *Appl. Numer. Math.*, **76** :1–18, 2014.

[2] A. Abdi and G. Hojjati. An extension of general linear methods. *Numer. Algor.*, **57** :149–167, 2011.

[3] J. C. Butcher and G. Hojjati. Second derivative methods with RK stability. *Numer. Algor.*, **40** :415–429, 2005.

[4] Z. Jackiewicz. *General linear methods for ordinary differential equations.* John Wiley & Sons Inc., Hoboken, NJ, 2009.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

TOWARDS HIGH-LEVEL PROGRAMMING OF STENCIL COMPUTATIONS ON PARALLEL SYSTEMS WITH MULTIPLE GPUS

STEFAN BREUER and MICHEL STEUWER and SERGEI GORLATCH

Departement of Mathematics and Computer Science

University of Münster, Germany

E-mail: {stefan.breuer, michel.steuwer, gorlatch}@uni-muenster.de

Stencil computations play an important role in a number of different application domains including time-intensive scientific simulations, image processing and others. Modern manycore architectures with Graphics Processing Units (GPUs) and other accelerators provide potentially tremendous computing power for challenging applications including stencil computations.

However, the current programming approaches for manycore architectures are low level, the most popular examples being OpenCL [1] and CUDA [2]. These approaches require the programmer to explicitly manage GPU's memory (including memory (de)allocations and data transfers to/from the system's main memory) and explicitly specify parallelism in the computation. This leads to lengthy, low-level, complicated and, thus, error-prone code. For multi-GPU systems, programming with CUDA and OpenCL is even more complex, as both approaches require an explicit implementation of data exchange between the GPUs, as well as disjoint management of each GPU, including low-level pointer arithmetics and offset calculations. When implementing stencil computations, additional challenges arise, like handling out-of-bound memory accesses and achieving high performance by making efficient use of the fast but small local GPU memory.

In this paper, we present our SkelCL [3] approach to high-level, manycore programming, and we describe how it simplifies stencil programming and achieves competitive performance on multi-GPU systems. SkelCL extends the OpenCL standard by three high-level mechanisms:

1) computations are easily expressed using pre-implemented parallel patterns (a.k.a. *skeletons*);
2) memory management is simplified using *container data types* for vectors and matrices;
3) data movement in multi-GPU systems are handled automatically by SkelCL's *(re)distribution mechanism*.

For stencil computations, we extend SkelCL with two specialized skeletons: MapOverlap for simple stencil computations, and Stencil for more complex, in particular iterative, stencil computations. We evaluate our approach using two real-world stencil computations and we compare our approach with related work.

REFERENCES

[1] . The OpenCl Specification. *Version 1.2.*, November 2012.

[2] . NVIDIA CUDA C Programming Guide . Version 5.5, July 2013.

[3] Michel Steuwer and Sergei Gorlatch. SkelCL: Enhancing OpenCL for high-level programming of multi-GPU systems. *In Malyshkin Victor, editor*, Parallel Computing Technologies - 12 th International Conference (PaCt 2013).Volume 7979 of Lecture Notes in Computer Science Springer Berlin Heidelberg 258–272, 2013

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

THE ROBUST FINITE VOLUMES APPROACH FOR COMPLICATED GEOMETRY OF DOMAINS WITH DISCONTINUOUS COEFFICIENTS OF MATERIAL PROPERTIES

ANDREJ BUGAJEV and NATALIJA TUMANOVA and OLGA SUBOČ

Dept. of Mathematical modelling, Vilnius Gediminas Technical University

Sauletekio av. 11, 10223 Vilnius, Lithuania

E-mail: `andrej.bugajev@vgtu.lt, natalija.tumanova@vgtu.lt, os@vgtu.lt`

We model heat transfer in middle and high voltage cables and around them. It is needed to answer many engineering questions, such as: maximal electric current for the cable, optimal cable parameters in certain circumstances, cable life expectancy calculation and other engineering questions.

The goal of our research is to propose the efficient approach for heat transfer mathematical modelling problem, when different configurations of electrical cables are investigated. The real problem can be described by this two dimensional heat transfer mathematical model

$$\begin{cases} c\rho\dfrac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + q, & t \in [0, t_{max}], x \in \Omega, \\ T(x,0) = T_b, & \text{when } x \in \Omega, \\ T(x,t) = T_b, & \text{when } x \in \partial\Omega, \\ [T] = [\lambda \nabla T] = 0 & \text{when } x \in S \end{cases} \tag{1}$$

where $x = (x_1, x_2)$, t is time, $T(x,t)$ is temperature, $\nabla \cdot$ is a divergence operator, ∇T is a gradient of T, $\lambda(x) > 0$ – thermal conductivity, $q(x,t,T)$ – source function. $\partial\Omega$ is a contour of Ω domain. $\rho(x) > 0$ – the mass density, $c(x) > 0$ – specific heat capacity. T_b, t_{max} – constants. λ, ρ, c are discontinuous and in neighboring domains can differ 1000 times, this makes the problem challenging. S is the contour of discontinuities of coefficients.

We apply Finite Volume Method (FVM) to problem (1). We use an open source software OpenFOAM which is oriented to FVM. The whole modelling domain consists of separate subdomains with different materials (soil, metal, isolation). To perform a correct discretization by finite volumes, we must adapt these volumes to the contours of subdomains. It gives a quite challenging a problem of so-called non-orthogonality errors. Different strategies exist to solve this issue. To perform tests for non-orthogonality and discontinuities of coefficients instead of problem (1) we solve stationary problem with one cable. First, we use a standard iterative method of non-orthogonality compensations, that is built-in inside OpenFOAM, and show that this approach is not optimal, since it affects the rate of FVM convergence negatively. Then we construct a triangulation that consists of acute triangles, that is achieved using software aCute, and we show that it eliminates the non-orthogonality issue. OpenFOAM doesn't support this technique, so we modify it and compile OpenFOAM version that fits our needs.

Subdomains are small comparing to the whole domain, so to perform a correct triangulation we propose to use an adaptive mesh with smaller triangles in particular subdomains. Also we discuss a strategy for selection adaptive time steps.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

NON UNIQUENESS OF BOUNDARY VALUE PROBLEMS ARISING IN MATHEMATICAL MODELLING OF BIOCEHEMICAL REACTIONS[1]

J.CEPITIS

Institute of Mathematics and Informatics, University of Latvia

Raina bulvāris 29, Rīga LV-1459, Latvija

E-mail: janis.cepitis@lu.lv

Mathematical modelling of fixed bed reactors using immobilized enzymes is important for a wide range of applications. The reaction rate R_p involved in the governing equations often has a non-Michaelis-Menten form which implies non uniqueness of boundary value problems (for example, see [1]). In general the function R_p is expressed like a rational function as a ratio of two polynomials [2]. The non uniqueness of solutions for such type boundary value problems will be considered.

REFERENCES

[1] J. Cepitis. Fifth Nordic-Baltic Agrometrics Conference Otepaa, Estonia, June 15–17,2005. In: *Mathematical Modelling of Biochemical Reactions for Certain Agriculture Production Processing*, U.Olsson and J.Sikk [Eds.], Swedish University of Agriculture Sciences, Uppsala, 2006, 35 – 40.

[2] International Union of Pure and Applied Chemistry. Symbolism and terminology in chemical kinetics. *Pure Appl.Chem.* , **53** 753-771, 1980.

[1]This work was partially supported by the grant 345/2012 of the Latvian Council of Science

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON PARALLELIZATION OF THE OPENFOAM-BASED SOLVER FOR THE HEAT TRANSFER IN ELECTRICAL POWER CABLES

RAIMONDAS ČIEGIS and VADIMAS STARIKOVIČIUS and ANDREJ BUGAJEV

Vilnius Gediminas Technical University

Saulėtekio al. 11, LT-10223, Vilnius, Lithuania

E-mail: {raimondas.ciegis, vadimas.starikovicius, andrej.bugajev}@vgtu.lt

Presently applicable standards for the design and installation of electrical power cables are often based on the analytical and heuristic formulas. Obviously, these formulas cannot accurately account for the various conditions under which the cables are installed and used. They estimate the cable's current-carrying capacity (ampacity) with significant margins to stay on the safe side. A more accurate mathematical modelling is needed to meet the latest technical and economical requirements and to elaborate improved (cost effective) design rules and standards.

When we need to deal with 3D transient mathematical models for the heat transfer in various media (metals, insulators, soil, water, air), only parallel computing technologies can allow us to get results in an adequate time. To solve numerically selected models, we develop our numerical solvers using the OpenFOAM package [1]. OpenFOAM is a free, open source CFD software package, which allows us to implement our own models, numerical schemes and algorithms, and to customize and extend existing OpenFOAM functionality to our needs. This package is well suited for solving systems of multiphysic models consisting of heat conduction, Euler, Navier-Sokes and porous media flow problems, which are important for our project. The important consequence of such an approach is that our numerical solvers can automatically exploit the parallel computing capabilities already available in the OpenFOAM package. Again we can adapt, extend and optimize the existing OpenFOAM algorithms and techniques for our needs.

First, we discuss the parallelization approach used in the OpenFOAM package. Next, we present and analyze the results on the efficiency and salability of the propsoed numerical solvers on various platforms: multicore CPUs, PC cluster, Linux and Windows operating systems.

REFERENCES

[1] OpenFOAM. http://www.openfoam.org

[2] General-Purpose computation on Graphics Processing Units. http://gpgpu.org

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON DIFFERENT TYPE SOLUTIONS OF THE BOUNDARY VALUE PROBLEMS

MARIA DOBKEVICH and FELIX SADYRBAEV[1]

Institute of Mathematics and Computer Science, University of Latvia

Raiņa bulvāris 29, Rīga LV-1459, Latvia

E-mail: marija.dobkevica@inbox.lv, felix@latnet.lv

We consider the Dirichlet problem

$$\begin{aligned} &x'' = f(t, x, x'), \\ &x(a) = A, \quad x(b) = B \end{aligned} \tag{1}$$

provided that all solutions are extendable to $[a, b]$. We define the type of a solution of BVP (1) as the number of zeros of $y(t)$ in (a, b), where $y(t)$ is a solution of the respective equation of variations

$$\begin{aligned} &y'' = f_x(t, x(t), x'(t))y + f_{x'}(t, x(t), x'(t))y' \\ &y(a) = 0, \quad y'(a) = 1. \end{aligned} \tag{2}$$

THEOREM 1. *Suppose that solutions of (1) are extendable to the interval $[a, b]$. If there exist a solution x_1 of problem (1) of m-type and a solution x_2 of n-type ($|m - n| \geq 2$), then there exist at least $|m - n| - 1$ more solutions of BVP (1).*

REFERENCES

[1] M. Dobkevich and F. Sadyrbaev, *Types of solutions and approximation of solutions of second order nonlinear boundary value problems.* Proceedings of ICNAAM. Amer. Inst. Phys. Publ., **1168** (1), 260–263 (2009).

[2] M. Dobkevich. *On Construction of Converging Sequences to Solutions of Boundary Value Problems.* Mathematical Modelling and Analysis **15** (2), 189–197 (2010).

[3] M. Dobkevich. *Non-Monotone Approximations Schemes for Solutions of BVP for ODE.* Proceedings of IMCS of University of Latvia, **11**, 83–89 (2011).

[1]This work is supported by ESF project 1DP/1.1.1.2.0/13/APIA/VIAA/045

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

COLLOCATION METHODS FOR PROBLEMS WITH FRACTIONAL DERIVATIVES

DANIELE FUNARO

Dipartimento di Fisica, Informatica e Matematica, University of Modena and Reggio Emilia

Via Campi 213/B, 41125 Modena (Italy)

E-mail: daniele.funaro@unimore.it

Approximations of fractional derivative operators by using spectral type methods are taken into account. A collocation procedure is studied for the Riemann-Liouville and the Caputo operators, when the approximation basis is connected to the set of Jacobi polynomials (see, e.g., [3], [4], [5], [6]). The technique is applied to second-order boundary-value problems also containing lower-order fractional derivatives. As suggested in [1] and [2] for other kind of problems, a way to achieve a better accuracy is to suitably choose the displacement of the collocation nodes in order to enlarge the dimension of the approximation space (superconsistency). This new grid may not be coincident with the one used to represent the solution. The results of some numerical experiments are discussed with the aim of showing how a wise set up of the grids can improve performances. This turns out to be particularly true when the second-order differential operator is dominated by the fractional one.

REFERENCES

[1] D. Funaro, Superconsistent discretizations, J. Scien. Comput., 17, 1-4 (2002), 67-79.

[2] D. Funaro, Superconsistent discretizations of integral type equations, Applied Numer. Math., 48 (2004), 1-11.

[3] X. Li, C. Xu, A space-time spectral method for the time fractional diffusion equation, SIAM J. Numer. Anal., 47 (2009), 2108-2131.

[4] W.Y. Tian, W. Deng, Y. Wu, Polynomial spectral collocation method for space fractional advection-diffusion equation, Numer. Meth. PDEs, 30-2 (2014), 514-535.

[5] M. Zayernouri, G.E. Karniadakis, Fractional Sturm-Liouville eigen-problems: theory and numerical approximation, J. Comput. Phys., 47 (2013), 2108-2131.

[6] M. Zayernouri, G.E. Karniadakis, Fractional spectral collocation method, SIAM J. Sci. Comput., 36-1 (2014), A40-A62.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ANALYTIC CONTINUATION OF TWISTED L-FUNCTIONS OF ELLIPTIC CURVES

VIRGINIJA GARBALIAUSKIENĖ

Institute of Informatics, Mathematics and e-studies, Šiauliai University

P. Višinskio str. 19, LT-77156 Šiauliai, Lithuania

E-mail: virginija@fm.su.lt

Suppose that E is an elliptic curve over the field of rational numbers with the discriminant $\Delta \neq 0$. The L-function $L_E(s)$, $s = \sigma + it$, of the curve E is defined, for $\sigma > \frac{3}{2}$, by

$$L_E(s) = \prod_{p|\Delta} \left(1 - \frac{\lambda(p)}{p^s}\right)^{-1} \prod_{p\nmid\Delta} \left(1 - \frac{\lambda(p)}{p^s} + \frac{1}{p^{2s-1}}\right)^{-1},$$

where $\lambda(p)$ is defined by $|E(\mathbb{F}_p)| = p+1-\lambda(p)$, and $|E(\mathbb{F}_p)|$ is the number of points of the reduction of the curve E modulo p (p is prime). Moreover, by the modularity conjecture proved in [1], $L_E(s)$ is an entire function.

Let N_E be the conductor of E, q be a prime number, $(N_E, q) = 1$, and let χ be a Dirichlet character modulo q. The twists $L_E(s, \chi)$ of the function $L_E(s)$ is defined, for $\sigma > \frac{3}{2}$, by

$$L_E(s,\chi) = \prod_{p|\Delta} \left(1 - \frac{\lambda(p)\chi(p)}{p^s}\right)^{-1} \prod_{p\nmid\Delta} \left(1 - \frac{\lambda(p)\chi(p)}{p^s} + \frac{\chi^2(p)}{p^{2s-1}}\right)^{-1}.$$

It turns out, that the function $L_E(s, \chi)$ also has analytic continuation to an entire function. Therefore, we may consider the function $L_E(s, \chi)$ for $\sigma > 1$. Suppose that $|\omega(p)| = 1$ for all primes p. Then, for $\sigma > 1$,

$$L_E(\sigma,\chi,\omega) = \prod_{p|\Delta} \left(1 - \frac{\lambda(p)\chi(p)\omega(p)}{p^\sigma}\right)^{-1} \prod_{p\nmid\Delta} \left(1 - \frac{\lambda(p)\chi(p)\omega(p)}{p^\sigma} + \frac{\chi^2(p)\omega^2(p)}{p^{2s-1}}\right)^{-1},$$

is a complex-valued random element on a certain probability space, and its distribution is the asymptotic limit measure of the function $L_E(\sigma+it, \chi)$. Also, the asymptotic distribution for $L_E(s, \chi)$ as $q \to \infty$ can be considered, and, for example, the main result of [2] remains valid for $\sigma > 1$.

REFERENCES

[1] C. Breuil, B. Conrad, F. Diamond and R. Taylor. On the modularity of elliptic curves over $\mathbb{Q}$: wild 3-adic exercises. *Journal of the American Mathematical Society*, **14** : 843–939, 2001.

[2] V. Garbaliauskienė, A. Laurinčikas and E. Stankus. Limit theorems for twist of L-functions of elliptic curves. *Lithuanian Mathematical Journal*, **50** (2): 187–197, 2010.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

POROELASTICITY. NUMERICAL DIFFICULTIES AND EFFICIENT MULTIGRID SOLUTION

FRANCISCO J. GASPAR

IUMA & Department of Applied Mathematics, University of Zaragoza

Pedro Cerbuna, 12, 50009, Zaragoza, Spain

E-mail: `fjgaspar@unizar.es`

The classical quasi-static Biot model for soil consolidation describes the time dependent interaction between the deformation of an elastic porous material and the fluid flow inside of it. This model can be formulated as a system of partial differential equations for the displacement of the solid and the pressure of the fluid.

On the one hand, due to a lack of compatibility between the boundary and initial conditions, a transient boundary layer appears in the pressure field. In such case standard approximation schemes produce non-physical oscillations in the numerical solution. Different stabilization strategies have been proposed to overcome this: from the use of staggered grids to stabilizations by adding a residual term to the weak form of the problem. We present and overview of some of the stabilization techniques that can be used when finite difference schemes are considered. We will show that these stabilized schemes result in numerical solutions without spurious oscillations, independently of the discretization parameters.

On the other hand, an important aspect in the numerical simulation of the problem is the efficient solution of the system of algebraic equations appearing after discretization. The resulting system is of saddle point type and we address its efficient solution by a geometric multigrid method. Different strategies can be used to design efficient multigrid algorithms for the poroelasticity equations, depending on the choice of the problem discretization.

Finally, some numerical experiments demonstrate the suitability of both the stabilized schemes and the multigrid solvers for this problem.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

TRAFFIC FLOW MODELLING VIA THE FORMALISM OF EVENT-SWITCHED PROCESSES

KONSTANTIN K. GLUKHAREV, ANDREY M. VALUEV, IVAN N. KALININ

Institute of Mechanical Engineering of RAS, Moscow State Mining University

Bardina str. 4, Moscow, Russia; Leninsky prospect 6, Moscow, Russia

E-mail: cafedravmtp@yandex.ru, valuev.online@gmail.com, pupssman@gmail.com

The paper presents generalization of the discrete flow model [1] based on description of car chains motion with a combination of ODEs and functional servo-links expressing a so-called "safe distance law" (SDL) that connects positions and velocities of subsequent vehicles.

A transportation system (TS) is a changing in time set of elements — moving or parked cars forming queues, commutators and regulators. Current state of the i-th TS element is described with a vector of state variables x_i and regime r_i, the latter changing only in moments $T_i(k_i)$ of reaching swithching manifolds when x_i may shift as well. Components of all x_i, k_i and r_i are united in vectors x, k and r. Dynamics between switches is subject to ODEs depending on control u_i

$$dx_i(t, k_i)/dt = f_i(r_i(k_i), x(t, k), u_i(t, k_i)), \quad i \in I(r(k)), \tag{1}$$

and restrictions must be satisfied

$$g_j(r_i(k_i), x(t, k), u_i(t, k_i)) \leq 0, j \in K_i(r_i(k_i)), \tag{2}$$

$$g_j(r_i(k_i), x(t, k)) < 0, j \in J_i(r_i(k_i)). \tag{3}$$

The value k_i changes to $k_i + 1$ at the earliest $t > T_i(k_i - 1)$ for which for some $j \in J_i(r_i(k_i))$

$$g_j(r_i(k_i), x(t, k)) = 0, \tag{4}$$

Then

$$r_i(k_i + 1) = R_j(r_i(k_i)), \quad x_i(T(k_i), k_i + 1) = X_j(k_i, x_i(T(k_i), k_i)). \tag{5}$$

There are dependences $U_i(r, x)$ so that setting $u_i(t, k_i) = U_i(r(k), x(t, k))$ in (1) while (3) is valid restrictions 2) would be satisfied.

Regarding (1)–(5) for flows on two intersecting single-lane roads with speed restriction by $v_{\max}$, we use for car dynamics $ds_i(t, k_i)/dt = u_i(t, k_i)$, $dv_i(t, k_i)/dt = u_i(t, k_i)$, $-b_{\max\ i} \leq u_i(t, k) \leq a_{\max\ i}$ and SDL as $s_i \geq s_{i+1} + S_{\text{safe}}(v_i, v_{i+1})$. There are four regimes for cars, $r = 1$ with $U_i(1) = a_{\max\ i}$, $r = 2$ with $U_i(2) = -b_{\max\ i}$, $r = 3$ — maintaining the maximum speed and r=4 — maintaining the safe distance, $U_i(4, r_i, s_i, s_{i-1}, \ldots v_i, v_{i-1}, \ldots)$ being determined recursively for a chain of cars in the same regime. Crossroad has no state variables and 3 regimes (free, occupied with a car going on the 1^{st} and the 2^{nd} road). Manifolds of switches are determined with equations $v_i = v_{\max}$, v_i=0, and some other expressing reaching the border of free or occupied crossroad and road ends.

REFERENCES

[1] Glukharev K.K., Ulyukov N.M., Valuev A.M., Kalinin I.N. On Traffic Flow on the Arterial Network Model. In: *Traffic and Granular Flow'11*, V.V. Kozlov et al.(Eds.), Springer, Berlin-Heidelberg, 2013, 399–412.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON OSCILLATORY NEHARI SOLUTIONS

ARMANDS GRITSANS[1] and FELIX SADYRBAEV[1,2]

[1] *Department of Mathematics, Daugavpils University*
Parades iela 1, Daugavpils LV-5400, Latvia
[2] *Institute of Mathematics and Computer Science, University of Latvia*
Raiņa bulvāris 29, Rīga LV-1459, Latvia
E-mail: armands.gricans@du.lv, felix@latnet.lv

The Nehari theory deals in particular with superlinear differential equations of the form

$$x'' = -q(t)|x|^{2\varepsilon}x, \quad \varepsilon > 0. \tag{1}$$

The Nehari numbers $\lambda_n(a,b)$ are minimal values of the functional

$$H(x) = \frac{\varepsilon}{1+\varepsilon}\int_a^b x'^2(t)dt$$

over the set of all $C^2([a,b],\mathbb{R})$ solutions of the boundary value problem (1),

$$x(a) = 0 = x(b), \quad x(t) \text{ has exactly } n-1 \text{ zeroes in } (a,b). \tag{2}$$

The BVP (1), (2) may have multiple solutions but not all of them are minimizers. Z. Nehari [1] posed the question is it true that there is only one minimizer associated with $\lambda_n(a,b)$. In [2] it was shown implicitly that there may be multiple minimizers associated with the number $\lambda_1(a,b)$. In the work [3] the example was constructed showing three solutions of the BVP (1), (2): two of solutions are non-even and one is an even function, besides two non-even solutions are minimizers.

Recently this problem was studied theoretically also by R. Kajikiya [4; 5]. In this talk we analyze Nehari numbers $\lambda_n(a,b)$ $(n > 1)$.

REFERENCES

[1] Z. Nehari. Characterictic values associated with a class of nonlinear second-order differential equations. *Acta Math.*, **105** (3), 1961, 141–175.

[2] F. Sadyrbaev. Solutions of an Emden-Fowler equation. *Differential Equations, Kluwer*, **25** (2), 1989, 560–565.

[3] A. Gritsans and F. Sadyrbaev. Characteristic numbers of non-autonomous Emden-Fowler type equations. *Math. Modelling and Analysis*, **11** (3), 2006, 243–252.

[4] R. Kajikiya. Non-even last energy solutions of the Emden-Fowler equation. *Proc. Amer. Math. Soc.*, **140** (4), 2012, 1353–1362.

[5] R. Kajikiya. Non-even positive solutions of the one-dimensional p-Laplace Emden-Fowler equation. *Appl. Math. Lett.*, **25** (11), 2012, 1891–1895.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

PSEUDO-PARABOLIC EQUATIONS SOLVING WITH THE COMBINED FINITE ELEMENT AND FINITE DIFFERENCE METHOD

GALINA HILKEVICA

Ventspils University College

Inzenieru iela 101, LV-3601, Ventspils, Latvia

E-mail: galina.hilkevica@venta.lv

Different physical processes such as fluid flow in fissured medium, heat conduction in composite medium, radioactive gas diffusion, etc. leads to pseudo-parabolic partial differential equations [1].

The typical pseudo-parabolic equation is

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} + \eta \frac{\partial^3 u(x,t)}{\partial x^2 \partial t}, \eta > 0. \tag{1}$$

Different kinds of solving techniques for the pseudo-parabolic equations have been developed during last years.

The main task of this article is to suggest the approach for pseudo-parabolic partial differential equation solution using the combined finite element and finite difference method. Numerical results received using combined finite element and finite difference method were compared with other numerical solutions and exact solution. It is shown that the suggested combined finite element and finite difference method has certain advantages comparing with other numerical methods.

REFERENCES

[1] R.E. Showalter and T. W. Ting. Pseudoparabolic partial differential equations. *SIAM Journal on Mathematical Analysis* , **1** (1):1–26, 1970.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

FINANCIAL TIME SERIES ANALYSIS AND PORTFOLIO MANAGEMENT THEORY

SERGEJS HILKEVICS

Ventspils University College

Inzenieru iela 101, LV-3601, Ventspils, Latvia

E-mail: sergejs.hilkevics@venta.lv

Financial time series $P(t)$ consist of three different components: a regular part $R(t)$ is determined by superposition of different scale periodic processes (e.g. economic cycles or seasonal price oscillations), chaotic part $H(t)$ is determined by chaotic properties of economy as dynamic system, stochastic part $S(t)$ is determined by random factors.

$$P(t) = \alpha R(t) + \beta H(t) + \gamma S(t), \quad \alpha + \beta + \gamma = 1$$

From forecasting point of view components $R(t)$, $H(t)$, $S(t)$ have completely different properties. Regular component $R(t)$ is predictable best of all, but on highly competitive financial markets contributions of regular oscillations are very small (0.5-3%). Stochastic component $S(t)$ is completely unpredictable in the case of repeated coin throws the previous history does not influence the result of next throw and there is no sense to analyse it for forecasting. Chaotic component predictability lies between $R(t)$ and $S(t)$ until certain time horizon $H(t)$ is predictable, but after that bifurcation makes forecasting impossible. In is easy to create the oscillation factor, which provides time series as specifically prepared mixtures of regular, chaotic and stochastic components with given α, β and γ, presented by superposition of several Van der Pol oscillators, Lorenz attractors and random generators. The ability to forecast the behaviour of such time series depends on α, β and γ values: the higher is weight of regular component, the better result of forecasting can be. Generated time series can be used for Markovitz optimal portfolios and compared with results received on real stock exchange data. The development of methods of forecasting of such kind of time series is very important for practical applications in portfolio theory.

REFERENCES

[1] B. LeBaron. Chaos and Nonlinear Forecastability in Economics and Finance. *Philosophical Transactions of the Royal Society of London*, 348A, 397-404 (1994).

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

DETERMINATION OF LOADING CAPABILITIES OF POWER CABLES

AUDRIUS ILGEVIČIUS

UAB Varikonta

Lazdynu g. 21 - 102, LT-04129 Vilnius, LITHUANIA

E-mail: audrius.ilgevicius@varikonta.lt

Electrical wires and cables for power transportation are well documented and standardized. But the design rules, which have to be applied for its application, are based more on practical experience than on precise measurements and calculations. In practice for existing cables higher current capacities are required because it is generally more difficult to install new cables in the urbanized areas. Therefore it is necessary the existing cables to operate in a more efficient way. The main aim of this Eureka Project "Mathematical modelling and optimization of electrical power cables for an improvement of their design rules" is the development of an numerical approach involving finite volume algorithm based on Crank-Nicholson approximation scheme for the loadability calculation of power cables. Analytical approaches are based on IEC 60287 standard can only be applied in homogeneous ambient conditions and on simple geometries. For example, formation of surrounding environment of a cable with several materials having different thermal properties, heat sources in the vicinity of the cable, non constant temperature limit values make the analytical solution difficult. In this case, only numerical approaches can be used.

Another issue for the engineers is the calculation of overload capability of power cables. This problem is not covered by the IEC 60287 standard. In order to maximise the utilisation of the existing cable network it can be beneficial to optimize the loadability of the cables and to take advantage of their overload capacity. Commonly used design criteria usually operate with a considerable safety margin, but do not allow overloading of the cables in order to allow for uncertainties in the load flow calculations. When the network loading becomes better known, it will be relevant to consider if the cables can be overloaded, how much and for how long. If the cables can be temporarily overloaded without a significant reduction of their service life, some grid investment can be avoided or postponed and hence money can be saved.

The term overload capacity is meant a current or power exceeding the ratings of the cable according to the manufacture's catalogue.

REFERENCES

[1] CIGRE Working Group B1.07. *Statistics on underground cable in transmission networks, Final report of CIGRE Working Group B1.07.* , 2006.

[2] G. J. Anders . Rating of Elektric Power Cables, Ampacity computations for Transmission, Distribution and Industrial Applications. *IEEE Transactions on Power Delivery*, 1997.

[3] IEC-Publ. 60287. Electric cables Calculation of the current rating (100% load factor). , 1994.

[4] IEC-Publ. 60853-2 . Calculation of the cyclic and emergency current rating of cables: Cyclic rating of cables greater than 18/30 (36) kV and emergency ratings for cables of all voltages. , 1989.

[5] Liess, H.-D.; Ilgevicius, A.. Analytical versus numerical calculations of physical problems. The benefit of its combination. *Mathematical Modelling and Analysis*, **(8), 4, 291–302** 2003.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON THE EXPLICIT FINITE DIFFERENCE SCHEMES FOR A PSEUDOPARABOLIC EQUATIONS WITH INTEGRAL CONDITIONS

JUSTINA JACHIMAVIČIENĖ [1] and MIFODIJUS SAPAGOVAS [2]

[1] *Vytautas Magnus University*
Vileikos 8, LT-44404, Kaunas, Lithuania
E-mail: `justina_jachimaviciene@yahoo.com`

[2] *Institute of Informatics and Mathematics, Vilnius University*
Akademijos 4, LT-08663, Vilnius, Lithuania
E-mail: `mifodijus.sapagovas@mii.vu.lt`

We consider the third order linear one-dimensional pseudoparabolic equation with a nonlocal conditions

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial}{\partial t \partial x^2} + f(x,t), \quad 0 < x < 1, \quad t > 0, \tag{1}$$

$$u(0,t) = \gamma_1 \int_0^1 u(x,t)\mathrm{d}x + \mu_1(t), \tag{2}$$

$$u(1,t) = \gamma_2 \int_0^1 u(x,t)\mathrm{d}x + \mu_2(t), \tag{3}$$

$$u(x,0) = \varphi(x). \tag{4}$$

where $\eta > 0$. An explicit conditionally consistent finite difference scheme is constructed and analyzed for this problem.

Note that, in the case $\eta \neq 0$, the third order derivative u_{txx} does not allow us to write an explicit two-layer finite difference scheme for equation (1). So we use a three-layer explicit finite difference scheme with a very special approximation of the derivative u_{txx} [1].

The stability of the finite difference scheme is investigated by analyzing a nonlinear eigenvalue problem. Also, we study the possibility to construct an explicit two-layer difference scheme for the case $\eta \neq 0$. The key for justifying such an approach lies in the combination of some implicit two-layer difference schemes with iterative processes.

REFERENCES

[1] J. Jachimavičienė, M. Sapagovas, A. Štikonas and O. Štikonienė. On the stability of explicit finite difference schemes for a pseudoparabolic equation with nonlocal conditions. *Nonlin. Anal. Model. Control*, **19** (2):225–240, 2014.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

STRONG STABILITY PRESERVING GENERAL LINEAR METHODS

ZDZISŁAW JACKIEWICZ

Arizona State University, School of Mathematical & Statistical Sciences

Tempe, Arizona 85287

E-mail: jackiewicz@asu.edu

We describe the construction of strong stability preserving (SSP) general linear methods (GLMs) for ordinary differential equations. This construction is based on the monotonicity criterion for SSP methods. This criterion can be formulated as a minimization problem, where the objective function depends on the Courant-Friedrichs-Levy (CFL) coefficient of the method, and the nonlinear constrains depend on the unknown remaining parameters of the methods. The solution to this constrained minimization problem leads to new SSP GLMs of high order and stage order. This is a joint work with Giuseppe Izzo from the University of Naples.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

FINITE DIFFERENCE METHOD FOR SOLVING A TWO–DIMENSIONAL PARABOLIC EQUATION WITH INTEGRAL BOUNDARY CONDITION

KRISTINA JAKUBĖLIENĖ

Kaunas University of Technology

Studentų St. 50, LT-51368, Kaunas, Lithuania

E-mail: kristina.jakubeliene@ktu.lt

We consider finite difference method for solving a linear two–dimensional parabolic equation with integral boundary condition

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} + f(x,y,t), \quad x,y \in \Omega, \tag{1}$$

$$u(x,y,t) = \iint\limits_{\Omega} K(x,y,\xi,\eta)u(\xi,\eta)d\xi d\eta + \mu(x,y,t), \quad x,y \in \Gamma, \tag{2}$$

$$u(x,y,0) = \varphi(x,y), \quad x,y \in \Omega, \tag{3}$$

where

$$\Omega = \{0 \leqslant x,y \leqslant 1\},$$
$$\Gamma = \{x=0, y \in [0,1]; x=1, y \in [0,1]; y=0, x \in [0,1]; y=1, x \in [0,1]\}.$$

The specific feature of this differential problem is that in the nonlocal condition the values of solution at contour points are associated with the integral of the solution in the whole domain Ω.

The method of alternating direction is applied for solving this differential problem. We obtain solutions of two one–dimensional difference problems applying nonlocal boundary conditions and then constructing two systems of linear algebraic equations. Numerical applications of these results are also studied.

REFERENCES

[1] M. Sapagovas and K. Jakuėlienė. Alternating direction method for two-dimensional parabolic equation with integral condition. *Nonlinear Anal. Model. Control*, **17** (1):91–98, 2012.

[2] K. Jakuėlienė and M. Sapagovas . On the stability of a difference scheme for a two-dimensionl parabolic equation with an integral condition. *Lith. Math. J*, **53** (3):311–323, 2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

NUMERICAL ANALYSIS OF SOME HEAT TRANSFER MODELS IN HIGH/MEDIUM VOLTAGE CABLES AND SURROUNDING AREA

GERDA JANKEVIČIŪTĖ and MEČISLAVAS MEILŪNAS and NATALIJA TUMANOVA

Department of Mathematical Modelling, Vilnius Gediminas Technical University

Saulėtekio av. 11, LT10223 Vilnius, Lithuania

E-mail: gerda.jankeviciute@vgtu.lt

We consider some heat transfer models in a cross-section of power cable and its surrounding area. Cables can be directly buried or laid out in buried ducts.

Today mathematical modelling of heat transfer in high voltage cables is obligatory for any power line design. It is necessary to calculate the temperature field of insulated cable in order to evaluate energy losses, longevity of the line and other parameters. Therefore models reflecting essential physical processes and properties of region of interest must be used, e.g. nonlinearity and nonhomogeneity must be taken into account.

We present some numerical results of steady state and transient simulation under various geometrical and physical conditions [1; 2; 3; 4].

REFERENCES

[1] D. Zhang. *Optimierung zwangsgekühlter Energiekabel durch dreidimensionale FEM-Simulationen.* University of Duisburg-Essen, Germany, 2009.

[2] J. Stammen. *Numerische Berechnung elektromagnetischer und thermischer Felder in Hochspannungskabelanlagen.* Shaker, Herzogenrath, 2001.

[3] W. Moutassem. *Configuration Optimization of Underground Cables inside a Large Magnetic Steel Casing for Best Ampacity.* University of Toronto, Canada, 2010.

[4] S. Trinks. *Einfluss des Wasser- und Wärmehaushaltes von Böden auf den Betrieb erdverlegter Energiekabel.* Selbstverl, Germany, 2010.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

UNIVERSALITY OF SOME FUNCTIONS RELATED TO DIRICHLET L-FUNCTIONS AND HURWITZ ZETA-FUNCTIONS

KESTUTIS JANULIS

Faculty of Mathematics and Informatics, Vilnius University

Naugarduko St. 24, LT - 03225, Vilnius, Lithuania

E-mail: `Kestutis.Janulis@gmail.com`

In the report, we discuss the results which are a continuotion of the paper [2]. In [2], a joint universality theorem for a collection consisting of Dirichlet L-functions $L(s,\chi)$ and Hurwitz zeta-functions $\zeta(s,\alpha)$, $0<\alpha\leq 1$, $s=\sigma+it$, has been proved, and the universality of the composite function $F(L(s,\chi_1),\dots,L(s,\chi_{r_1}),\zeta(s,\alpha_2),\dots,\zeta(s,\alpha_{r_1}))$ for one class of operators F has been obtained. In the report, we consider other classes of operators F. We give one example.

Let $D=\{s\in\mathbb{C}:\frac{1}{2}<\sigma<1\}$. Denote by $H(D)$ the space of analytic functions on D endowed with the topology of uniform convergence on compacta, and let

$$S=\{g\in H(D):g(s)\neq 0 \text{ or } g(s)\equiv 0\}.$$

For arbitrary distinct complex numbers $a_1,\dots,a_r$, define

$$H_r(D)=\{g\in H(D):(g(s)-a_j)^{-1}\in H(D), j=1,\dots,r\}.$$

Theorem 1. *Suppose that $\chi_1,\dots,\chi_{r_1}$ are pairwise non-equivalent Dirichlet characters, the numbers $\alpha_1,\dots,\alpha_r$ are algebracally independent over the field of rational numbers, and that $F:H^{r+r_1}(D)\to H(D)$ is a continuous operator such that $F(S^{r_1}\times H^{r_2}(D))\supset H_r(D)$. If $r=1$, let $K\subset D$ be a compact subset with connected complement, and $f(s)$ be a continuous $f(s)\neq a_1$ function on K which is analytic in the interior of K. If $r\geq 2$, let $K\subset D$ be on arbitrary compact subset, and $f(s)\in H_r(D)$. Then, for every $\varepsilon>0$,*

$$\liminf_{T\to\infty}\frac{1}{T}meas\{\tau\in[0;T]:\sup_{s\in K}|F(L(s+i\tau,\chi_1),\dots,L(s+i\tau,\chi_{r_1}),$$
$$\zeta(s+i\tau,\alpha_1),\dots,\zeta(s+i\tau,\alpha_{r_2}))-f(s)|<\varepsilon\}>0.$$

REFERENCES

[1] K. Janulis, A. Laurinčikas. Joint universality of Dirichlet L-functions and Hurwitz zeta-functions. *Annales Univ. Sci. Budapest., Sect. Comp.*, **39:** 203 - 214, 2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON THE MUTH DISTRIBUTION

P. JODRÁ[1] and M.D. JIMÉNEZ-GAMERO[2] and M.V. ALBA-FERNÁNDEZ[3]

[1]*Dpto. de Métodos Estadísticos, Universidad de Zaragoza*
María de Luna 3, 50018 Zaragoza, Spain
[2]*Dpto. de Estadística e Investigación Operativa, Universidad de Sevilla*
Avd. Reina Mercedes s.n, 41012 Sevilla, Spain
[3]*Dpto. de Estadística e Investigación Operativa, Universidad de Jaén*
Paraje Las Lagunillas s.n., 23071 Jaén, Spain
E-mail: pjodra@unizar.es, dolores@us.es, mvalba@ujaen.es

Muth [3] introduced a continuous probability distribution X with cumulative distribution function (cdf)

$$F(x) := P(X \leq x) = 1 - \exp\Big(\alpha x - \frac{1}{\alpha}\left(e^{\alpha x} - 1\right)\Big), \quad x > 0, \tag{1}$$

where the parameter $\alpha \in (0, 1]$. In this work, we study some statistical properties of the Muth distribution. More specifically, we show that the mode of X can be expressed as a function of the golden ratio ($\varphi := (1 + \sqrt{5})/2$), the non-central moments $E[X^k]$ can be expressed in terms of the generalized integro-exponential function (cf. Milgram [2]) and the quantile function $Q(u)$ can be given in closed-form in terms of the Lambert W function, so the Muth distribution has the variate generation property (cf. Leemis and McQueston [1]). We summarize these results in the following proposition.

PROPOSITION 1. *Let X be a random variable with cdf* (1)*, then*

(i) $\mathrm{mode}(X) = (1/\alpha)\log(\alpha\varphi^2)$ *if* $1/\varphi^2 < \alpha \leq 1$ *and* $\mathrm{mode}(X) = 0$ *if* $0 < \alpha \leq 1/\varphi^2$.

(ii) $E[X^k] = (\Gamma(k+1)/\alpha^k)e^{1/\alpha}E_0^{k-1}(1/\alpha)$, $k = 1, 2, \ldots$. *In particular,* $E[X] = 1$ *and* $E[X^2] = (2/\alpha)e^{1/\alpha}E_1(1/\alpha)$, *where* E_1 *denotes the best-known exponential integral function.*

(iii) $Q(u) = \dfrac{1}{\alpha}\log(1-u) - \dfrac{1}{\alpha^2} - \dfrac{1}{\alpha}W_{-1}\left(\dfrac{u-1}{\alpha e^{1/\alpha}}\right)$, $0 < u < 1$, *where* W_{-1} *denotes the negative branch of the Lambert* W *function.*

In addition, a simulation study is carried out to compare weighted and unweighted least squares methods and the maximum likelihood method for estimating the parameter α. To this end, random samples from the Muth distribution can be computer-generated in a straightforward manner by virtue of Proposition 1(iii).

REFERENCES

[1] L.M. Leemis and J.T. McQueston. Univariate distribution relationships. *Am. Stat.*, **62** (1): 45–53, 2008.

[2] M.S. Milgram. The generalized integro-exponential function. *Math. Comp.*, **44** (170): 443–458, 1985.

[3] J.E. Muth. Reliability models with positive memory derived from the mean residual life function. In: The Theory and Applications of Reliability, C.P. Tsokos and I. Shimi (Eds.), Academic Press, New York., 1977, 401–435.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON THE STRUCTURE OF SPECTRUM OF DIFFERENTIAL OPERATOR WITH VARIABLE COEFFICIENTS AND NONLOCAL BOUNDARY CONDITIONS

Ž. JOKŠIENĖ[1] and R. ČIUPAILA[2] and M. SAPAGOVAS[3]

[1] *Lithuanian University of Health Sciences, Vytautas Magnus University,* [2] *Vilnius Gediminas Technical University,* [3] *Vilnius University, Institute of Mathematics and Informatics*

[1]Eiveniu 4, LT-50009, Kaunas, Lithuania, [2]Saulėtekio al. 11, LT-10223, Vilnius, Lithuania, [3]Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: zivile.je@gmail.com, regimantas.ciupaila@vgtu.lt

E-mail: mifodijus.sapagovas@mii.vu.lt

The eigenvalue problem for a two-dimensional elliptic operator with integral conditions

$$\frac{\partial}{\partial x}\Big(p_1(x)\frac{\partial u}{\partial x}\Big) + \frac{\partial}{\partial y}\Big(p_2(y)\frac{\partial u}{\partial y}\Big) + \lambda u = 0, \qquad (x,y) \in D = \{0,x,y,1\}, \tag{1}$$

$$u(0,y) = \gamma_1 \int_0^1 u(x,y)\mathrm{d}x, \qquad u(1,y) = \gamma_2 \int_0^1 u(x,y)\mathrm{d}x, \qquad u(x,0) = u(x,1) = 0. \tag{2}$$

is considered. The eigenvalue problem for differential operators with the variable coefficients and nonlocal boundary conditions is a less investigated problem. However, the structure of differential and difference operators with nonlocal conditions is very important for the investigation of the stability of difference schemes and convergence of iterative methods [1, 2].

Under the condition $\gamma_1 = \gamma_2 = 0$, all the eigenvalues for the problem (1)–(2) are real and positive. The main purpose of the presentation is to investigate, when the eigenvalue $\lambda = 0$ and $\lambda <$ could exist depending on the values of γ_1, γ_2 and the properties of coefficients $p_1(x), p_2(y)$.

In the presentation, it is proven that the existence of eigenvalues $\lambda = 0$ and $\lambda <$ depends on the properties of $p_1(x)$ such as monotonicity or symmetricity, regarding the point $x = 1/2$, and the meaning of $\int_0^1 \frac{\mathrm{d}x}{p_1(x)}$. The reasons, on which the number of negative eigenvalues depends, were considered. The results of the presentation are the generalization and evolvement of the results earlier obtained for the one-dimensional problem [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big(p(x)\frac{\mathrm{d}u}{\mathrm{d}x}\Big) + \lambda u = 0, \qquad 0 < x < 1, \qquad u(0) = \gamma_1 \int_0^1 u(x)\mathrm{d}x, \qquad u(1) = \gamma_2 \int_0^1 u(x)\mathrm{d}x.$$

REFERENCES

[1] Y.Wang. Solution to nonlinear elliptic equations with nonlocal boundary condition. *Electr. J. Differ. Eq.*, **2002** 5: 1–16, 2002.

[2] M.Sapagovas, A.Štikonas and O.Štikonienė. Alternating direction method for the Poisson equation with variable weight coefficients in an integral condition. *Differ. Equat.*, **47** 8: 1176–1187, 2011.

[3] Ž.Jokšienė, R.Čiupaila and M.Sapagovas. The eigenvalues problem for one-dimensional differential operator with variable coefficient and nonlocal condition. *Lith. Math. J.*, 2014 (to appear).

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON VALUE DISTRIBUTION OF ZETA-FUNCTIONS ATTACHED TO CERTAIN CUSP FORMS

ROMA KAČINSKAITĖ and DAIVA KORSAKIENĖ

Department of Mathematics, Šiauliai University

P. Višinskio str. 19, LT-77156 Šiauliai, Lithuania

E-mail: r.kacinskaite@fm.su.lt, korsakiene@fm.su.lt

Let $F(z)$ be a holomorphic cusp form of weight κ of the full modular group $SL(2,\mathbb{Z})$. Suppose that $F(z)$ is a normalized eigenform having Fourier series expansion

$$F(z)=\sum_{m=1}^{\infty} c(m)e^{2\pi imz}, \quad c(1)=1.$$

The zeta-function attached to normalized eigenform, for $\sigma > \frac{\kappa+1}{2}$, is defined by absolutely convergent Dirichlet series with coefficients $c(m)$

$$\varphi(s,F)=\sum_{m=1}^{\infty} c(m)m^{-s},$$

and it is analytically continuable to an entire function.

In [1], A. Laurinčikas and K. Matsumoto proved the joint universality of zeta-functions attached to normalized eigenforms. To obtain the joint universality a limit theorem for the collection of functions $\varphi(s,F_1),...,\varphi(s,F_n)$, when cusp forms $F_1(z),...,F_n(z)$ satisfy some additional conditions, is used.

Following the ideas in [1], we discuss on joint discrete value distribution of zeta-functions attached to certain cusp forms.

First author is partially supported by the European Commission within the 7th Framework Programme 2011–2015 project INTEGER (INstitutional Transformation for Effecting Gender Equality in Research), Grant Agreement No. 266638.

REFERENCES

[1] A. Laurinčikas and K. Matsumoto. The joint universality of zeta-functions attached to certain cusp forms. *Fiz. Mat. Fak. Moksl. Semin. Darb.*, **5** :58–75, 2002.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON NUMERICAL SIMULATION OF BOUNDARY-VALUE PROBLEMS WITH PERIODICAL BOUNDARY CONDITIONS [1]

H. KALIS and O.LIETUVIETIS

Institute of Mathematics and Informatics, University of Latvia

Raina bulvāris 29, Rīga LV-1459, Latvija

E-mail: kalis@lanet.lv, ojars.lietuvietis@lu.lv

In this talk, the finite difference scheme (FDS) for approximating periodic function's derivatives in a $2n+1$ point stencil is studied, obtaining higher order accuracy approximation. This method in the uniform grid with N mesh points is used to approximate the differential operator of the second and the first order derivatives in the space, using the multi-point stencil. The described methods are applicable for various mathematical physics problems involving periodic boundary conditions(PBC). The solutions of some linear problems for parabolic type partial differential equations (PDE) with PBCs are obtained, using the method of lines (MOL) to approach the PDEs in the time and the discretization in space applying the FDS of different order of the approximation and finite difference scheme with exact spectrum (FDSES) [2].

These methods are compared with the global approximations method [1], which is based on using such differentiation matrices A for derivatives and nonuniform grid with Chebyshev nodes. The linear heat transfer equations can be written in the following form:

$$u_t(x,t) = k(x)(u(x,t))_{xx} + p(x)(u(x,t))_x + q(x)u(x,t) + f(x), \;\; u(x,0 = u_0(x),$$

where $k(x)$, $p(x)$, $q(x)$, $u_0(x)$ are real functions, $x \in (0,L)$, $t > 0$ are the space and time variables, L is the period, $u = u(x,t)$ is the unknown function (for ODE we have boundary value problem with $u = u(x)$). Note, that the similarly system of PDE is considered, where k, p, q are matrices, u is column-vector. We have the discrete equations $(x_j = jh, Nh = L, j = \overline{1,N})$as a system of ODEs in following form:

$$\dot{U} = K(A_{2,n}(U) + P(A_{1,n}(U) + Q(U) + F, \;\; U(0) = U_0,$$

where $A_{1,n}, A_{2,n}$ are N-th order circulant matrices, $U = U(t)$, F, $\dot{U}$, U_0 are the column-vectors of the N order, K, P, Q are N-th order diagonal matrices.

REFERENCES

[1] T. Cirulis, O.Lietuvietis. Degenarete matrix method with Chebyshev nodes for solving nonlinear systems of differential equations . *Mathematical Modelling and Analysis* , **4** 454.1999, 51-57

[2] H.Kalis and A.Buikis . Method of lines and finite difference schemes of exact spectrum for solution the hyperbolic heat conduction equation . *Mathematical Modelling and Analysis* , **16** 2 .2011, 220-232

[1] This work was partially supported by the grant 623/2014 of the Latvian Council of Science

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON SIMULATION OF VISCOUS INCOMPRESSIBLE ELECTRICALLY CONDUCTING FLOW AND TEMPERATURE AROUND PERIODICALLY PLACED CYLINDERS[1]

H. KALIS and M. MARINAKI

Institute of Mathematics and Informatics, University of Latvia

Raina bulvāris 29, Rīga LV-1459, Latvija

E-mail: `kalis@lanet.lv, maksims.marinaki@lu.lv`

We consider 2D stationary boundary value problems for the system of magnetohydrodynamic (MHD) equations . The viscous electrically conducting incompressible liquid is moving between infinite cylinders placed periodically. We analyze the 2D flow and temperature around these cylinders in homogeneous external magnetics field, depending on direction of the gravitation. The distribution of electromagnetic fields, forces , velocity and temperature around a cylinders has been calculated using finite difference and finite elements methods.
The liquid motion can be described by the dimensionless stationary Navier-Stokes and heat transfer equations in the Cartesian coordinates (x, y) :

$$\begin{cases} -\zeta V_y = -\dfrac{\partial \bar{p}}{\partial x} - Re^{-1}\dfrac{\partial \zeta}{\partial y} - S\sin(\alpha) j_z + \dfrac{Gr}{Re^2} T \sin(\beta), \\ \zeta V_x = -\dfrac{\partial \bar{p}}{\partial y} + Re^{-1}\dfrac{\partial \zeta}{\partial x} + S\cos(\alpha) j_z - \frac{Gr}{Re^2} T\cos(\beta), \\ \dfrac{\partial (V_x)}{\partial x} + \frac{\partial (V_y)}{\partial y} = 0, \\ V_x \dfrac{\partial T}{\partial x} + V_y \dfrac{\partial T}{\partial y} = Pe^{-1}\Delta T + \dfrac{K_T}{Pe} j_z^2, \end{cases} \tag{1}$$

where $j_z = E_z - V_y\cos(\alpha) + V_x \sin(\alpha)$ is the axial component for the electric current density, Δ is Laplace operator, $\bar{p} = p + 0.5\mathbf{V^2}$, p is the pressure, $\zeta = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}$ is the vorticity function, $Re = \frac{U_0 L_0}{\nu}$, $S = \frac{\sigma B_0^2 L_0}{\rho U_0}$, $Gr = \frac{\beta_t g (T_w - T_0) L_0^3}{\nu^2}$ are Reynolds, Stewart and Grashof numbers, $Pe = PrRe$, $Pr = \frac{\nu \rho c_p}{k}$, $K_T = \frac{\sigma B_0^2 L_0^2 U_0^2}{k T_w - T_0}$ are Prandtl number and heat source parameters, $\sigma, \rho, \nu, k, c_p, g, \beta_t$ are electrical conductivity, fluid density, kinematic viscosity, heat conductivity, specific heat, acceleration due to gravity, volumetric coefficient of thermal expansion, $B_0, U_0, L_0, P_0 = U_0^2\rho, T_0, T_w$ are the characteristic uniform magnetic field strength, velocity, lengths,pressure, initial fluid temperature and temperature on the cylinders , $E_z = const$ is the azimuthal component of the electric field E, α is the angle between the Ox- axis and direction of the induction vector, β is the angle between Oy- axis and direction of the gravitation vector.

The hydrodynamical stream function ψ can be determined via formulas $V_x = \frac{\partial \psi}{\partial y}, V_y = -\frac{\partial \psi}{\partial x}$ and the pressure is eliminated from (1) .

[1]This work was partially supported by the grant 623/2014 of the Latvian Council of Science

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

CLASSICAL SOLUTION OF THE BOUNDARY PROBLEMS FOR THE NONSTRONGLY HYPERBOLIC EQUATIONS OF THE SECOND ORDER

VICTOR I. KORZYUK, ELENA S. CHEB

Institute of Mathematics of National Academy of Science, Belarussian State University

11 Surganova St., 220072; 4 Nezavisimosti Ave., 220030 , Minsk, Belarus

E-mail: Korzyuk@bsu.by; Cheb@bsu.by

The classical solution to boundary value problems for nonstrongly hyperbolic equation of the second order. It is known that classical solution is the basis of the theory of numerical methods for boundary value problems for partial differential equations. In the report, the method of characteristics [1] is considered to construct the classical solution to boundary value problems for general hyperbolic equations of the second order.

In the half-string $Q = (0, \infty) \times (0, l)$ the linear partial differential equation with independent variables t, x is considered

$$(\partial_t - a\partial_x + b_1)\,(\partial_t - a\partial_x + b_2)\,u(t, x) = f(t, x), \tag{1}$$

where $a > 0, b_1, b_2 \geq 0, b_1 \neq b_2$.

The Cauchy conditions

$$u|_{t=0} = \varphi(x), \quad \partial_t u\Big|_{t=0} = \varphi_j(x), \quad x \in (0, l). \tag{2}$$

and the boundary conditions

$$u|_{x=0} = \mu_1(t), \quad t \in (l/a, \infty), \quad u|_{x=l} = \mu_2(t), \quad t \in (0, \infty); \tag{3}$$

or

$$\partial_x u|_{x=0} = \mu_1(t), \quad t \in (l/a, \infty), \quad \partial_x u|_{x=l} = \mu_2(t), \quad t \in (0, \infty); \tag{4}$$

or

$$(\partial_x u + u)|_{x=0} = \mu_1(t), \quad t \in (l/a, \infty), \quad (\partial_x u + u)|_{x=l} = \mu_2(t), \quad t \in (0, \infty) \tag{5}$$

are added to the equation (1).

Conditions (3)–(5) are chosen so that each of the boundary value problems was well-posed. Matching conditions for the functions φ, ψ, μ_1, μ_2 providing the uniqueness of the classical solution are obtained. It is noticed that formulation of well-posed problems in the sense of Hadamard for nonstrongly hyperbolic equation increases demands on the smoothness of the initial data as compared with strictly hyperbolic equation.

REFERENCES

[1] Korzyuk V. I., Cheb E. S. and Karpechina A. A. Classical solution of the first boundary problem in the half-string for the linear hyperbolic equations of the second order. *Trudy IM NAN Belarusi.*, **20** (2), 2012, 64 – 67.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON PROBLEM DEFINITION FOR PARTIAL DIFFERENTIAL EQUATIONS IN CASE OF CLASSICAL AND NUMERICAL SOLUTIONS

V. I. KORZYUK, I. S. KOZLOVSKAYA

Belarusian State University

4, Nezavisimosti av., Minsk, Belarus, 220030

Institute of Mathematics of National Academy of Science

11, Surganova St., Minsk, Belarus, 220072

E-mail: `Korzyuk@bsu.by`

Classical solutions for various boundary value problems and problems with integral conditions for partial differential equations were tackled by authors ([1–3] and other). We present analytical solutions in case of two independent variables for the following problems:

- Cauchy problem for hyperbolic equation with constant coefficients. Its operator is the composition of first-order operators;
- mixed boundary value problems for a second-order hyperbolic equation;
- mixed boundary value problems for a bi-wave equation;
- mixed boundary value problems for a wave equation with integral boundary conditions;
- a wave equation with integral conditions in the domain of the solution;
- control problems of Cauchy or boundary value conditions when values of the unknown function or its derivatives are specified inside the domain of the solution.

We show that classical solutions defined in the whole domain of the equation depend on homogeneous compatibility conditions for given boundary conditions at common angular points of the boundary. These compatibility conditions are necessary and sufficient. When these conditions become inhomogeneous, well-posed problems need to be defined in a different way. For example, they can be posed as conjugation problems with conjugation conditions on proper characteristics inside the domain of the unknown function.

On the other hand, there are well-known numerical methods for solving boundary value problem and other kinds of problems, which assume the existence of classical solutions. Generally, the literature on numerical methods does not discuss or does not stress the importance of compatibility conditions for given boundary conditions. However, this is crucial for full and correct definition of various problems under discussion and for the correct usage of corresponding numerical methods.

REFERENCES

[1] V. I. Korzyuk, I. S. Kozlovskaya. Article in journal. *Proceedings of the Institute of Mathematics NAS of Belarus*, **19** (1):62–70, 2011.

[2] Korzyuk V.I., Erofeenko. V.T., Sheika J.V.. Article in journal. *Mathematical Modelling and Analysis*, **17** (3): 309–329, 2012.

[3] Korzyuk V.I., Kozlovskaya I.S.. Article in proceedings. In: *Proc. of the Conference Com-puter Algebra Systems in Teaching and Research. Differential Equations and Symbolic Computation, Siedlce, Poland, 2013*, Gadomski L., Jakubiak M., Prokopenya A.N. (Eds.), 2013, 53 – 65.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

EXPERT ESTIMATES AVERAGING BY CONSTRUCTING INTUITIONISTIC FUZZY TRIANGLES

ALEKSANDRAS KRYLOVAS[1] and NATALJA KOSAREVA[2]

[1] *Department of Mathematical Modelling, Mykolas Romeris University*

Ateities str. 20, LT08303, Vilnius, Lithuania

[2] *Department of Mathematical Modelling, Vilnius Gediminas Technical University*

Sauletekio av. 11, LT10223, Vilnius, Lithuania

E-mail: `krylovas@mruni.eu, Natalja.Kosareva@vgtu.lt`

The problem of ranking (sorting) of m alternatives $A_1, A_2, \dots, A_m$ was considered when experts evaluate each alternative according to k criteria $r_1, r_2, \dots, r_k$. Evaluation information is given by matrix $D_{m\times k} = (d_{ij})$, where $d_{ij} \in [0;1]$ are real numbers. Suppose that criteria r_j have weights w_j : $0 < w_j < 1, \sum_{j=1}^{k} w_j = 1$. Then functions of arithmetic, geometric or other weighted averages [3] $M(A_i) = M(d_{i1}, d_{i2}, \dots, d_{ik})$ are constructed for decision making.

It was shown [2] that in many cases intuitionistic fuzzy approach lets to obtain better results in solving Multiple Criteria Decision Making problems than dealing with real numbers and fuzzy numbers.

We investigated generalization of this scheme when there are evaluation matrices of several experts and this information is aggregated in the form of triangular intuitionistic fuzzy numbers. The notion of intuitionistic fuzzy numbers was introduced by K. Atanassov [1] and it is defined as follows:

$$T = ([a^L, a^M, a^U], [b^L, b^M, B^U]), \tag{1}$$

here $0 \leqslant a^L \leqslant a^M \leqslant a^U \leqslant 1, \quad 0 \leqslant b^L \leqslant b^M \leqslant b^U \leqslant 1, \quad a^U + b^U \leqslant 1$. Numbers a^L, a^M, a^U express compliance level and b^L, b^M, b^U – noncompliance level of the certain alternative for a given criterion.

Fuzzy triangles were constructed by (1) with different uncertainty levels, expert's decision matrices and numbers of experts which varied from 2 to 5. Method for construction of expert's decision probability matrices for the special case when experts evaluate alternatives A_i according to criteria r_j with scores $d_{ij} \in \{1, 2, 3\}$ is proposed.

The results were obtained by performing Monte Carlo simulations and probabilities of errors compared for arithmetic, geometric, fuzzy arithmetic and fuzzy geometric averages.

REFERENCES

[1] K. Atanassov. *Intuitionistic Fuzzy Sets. Theory and Applications.* Heidelberg; New York: Physica-Verlag HD, 1999.

[2] N. Kosareva and A. Krylovas. Comparison of accuracy in ranking alternatives performing generalized fuzzy average functions. *Technological and Economic Development of Economy*, **19** (1):162–187, 2013.

[3] X. Zhang and P. Liu. Method for aggregating triangular fuzzy intuitionistic fuzzy information and its application to decision making. *Technological and Economic Development of Economy*, **16** (2):280–290, 2010.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

RESONANCE INTERACTION OF ACOUSTIC PERIODIC WAVES IN NON-IDEAL GAS

ALEKSANDRAS KRYLOVAS and OLGA LAVCEL-BUDKO

Dept. of Mathematical Modelling, Vilnius Gediminas Technical University

Sauletekio av. 11, 10223 Vilnius, Lithuania

E-mail: akr@fm.vgtu.lt, olecka@gmail.com

We consider differential equations of gas dynamics, when the thermal conductivity and viscosity coefficients equal zero [2].

The system is supplemented with equations of state for the gas pressure and the total energy; these equations do not depend on the properties of gas, i.e. are of a general form.

We will discuss small perturbations of gas steady state, resulting interaction of acoustic waves. After a complete asymptotical analysis of the system we construct asymptotical approximation for the resonance interaction of acoustic periodic waves, which is equally appropriate in the long time interval [3].

The system of averaged equations is investigated. The additional analysis of system coefficients allows determining the resonance occurring condition – the relation between gas pressure and temperature at the initial time.

Real gas approximately satisfies the ideal gas equation of state at a sufficiently high temperature and low density. Several hundred empirically determined state equations are used for more precise expression of real gas state. The van der Waals equation is the simplest and best known one; two parameters of this equation depend on properties of gas.

We investigate the ideal and non-ideal gas and determine the relationship between the parameters, which causes the vanishing of resonance in non-ideal gas. All the results we present allow to generalize the case of ideal polytropic gas, discussed in [1].

REFERENCES

[1] A. Krylovas, R. Čiegis. Asymptotical analysis of one dimensional gas dynamics equations. *Math. Model. Anal.*, **6** (1):117–128, 2001.

[2] B. L. Rozhdestvenskii, N. N. Yanenko . *Systems of Quasilinear Equations and their Application to Gas Dynamics*. Nauka, Moscow , 1978.

[3] A. Krylovas, O. Lavcel-Budko . Asymptotical approximation of one dimensional gas dynamics problem. *// Lietuvos matematikos rinkinys. LMD darbai. Vilnius: VU Matematikos ir informatikos institutas*, **54** (B):48–53, 2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

NUMERICAL SOLUTION USING SPECIAL LAYER ADAPTED MESHES FOR SINGULARLY PERTURBED PROBLEMS

VIVEK KUMAR and DEEPTI KAUR

Department of Applied Mathematics, Delhi Technological University and Department of Mathematics, University of Delhi

DTU, Delhi - 110042 and DU, Delhi -110007, INDIA

E-mail: vivekkumar.ag@gmail.com, deeptimaths15@gmail.com

The objective of this paper is to present a comparative study of simple upwind finite difference methods on various non-uniform meshes existing in the literature for resolving the boundary layer of singularly perturbed problems. Our exposition begins with the Bakhvalov mesh [1; 2] and its modification using padé approximation, then continues with piecewise uniform Shishkin mesh and to the most recent W-grid using Lambert W-function [3]. A new kind of mesh of Shishkin type that incorporate the idea of Lambert W-function has also been proposed and using this mesh, the method gives better results as compared to the results using the recent W-grid. For various meshes, the computed solution is uniformly convergent with respect to the small perturbation parameter. Numerical results on some test problems are presented which validate the theoretical considerations.

REFERENCES

[1] N.S. Bakhvalov. Towards optimization of methods for solving boundary value problems in the presence of boundary layer. *Zh. Vychisl. Mater. Fiz.*, **9,** 841–859, 1969.(in Russian)

[2] T. Linß. Layer-adapted meshes for convection - diffusion problems. *Comput. methods Appl. Mech. Engrg.*, **192,** 1061–1105, 2003.

[3] H.G. Roos, L. Teofanov, Z. Uzelac. A modified Bakhvalov mesh. *Applied Mathematics Letters*, **31,** 7–11, 2014.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

DISCRETE UNIVERSALITY OF THE HURWITZ ZETA-FUNCTION

ANTANAS LAURINČIKAS

Department of Mathematics and Informatics, Vilnius University

Naugarduko 24, LT-03225 Vilnius, Lithuania

E-mail: antanas.laurincikas@mif.vu.lt

Let $s = \sigma + it$ be a complex variable, and α, $0 < \alpha \leq 1$, be a fixed parameter. The Hurwitz zeta-function $\zeta(s, \alpha)$ with parameter α is defined, for $\sigma > 1$, by the Dirichlet series

$$\zeta(s, \alpha) = \sum_{m=0}^{\infty} \frac{1}{(m + \alpha)^s},$$

and is analytically continued to the whole complex plane, except for a simple pole at the point $s = 1$ with residue 1.

It is well known that the function $\zeta(s, \alpha)$ for some classes of the parameter α is universal, that is, its continuous shifts $\zeta(s + i\tau, \alpha)$, $\tau \in \mathbb{R}$, or discrete shifts $\zeta(s + imh, \alpha)$, $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $h > 0$ is a fixed number, approximate every analytic function. Let $D = \left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$. Denote by $\mathcal{K}$ the class of compact subsets of D with connected complements, and by $H(K)$, $K \in \mathcal{K}$, the class of continuous functions on K which are analytic in the interior of K. In discrete case the following statement is known.

THEOREM 1. *Suppose that α is transcendental or rational number $\neq 1, \frac{1}{2}$. Let $K \in \mathcal{K}$ and $f(s) \in H(K)$. In the case of rational α, let the number $h > 0$ be arbitrary, while in the case of transcendental α, let $h > 0$ be such that $\exp\left\{\frac{2\pi}{h}\right\}$ is a rational number. Then, for every $\varepsilon > 0$,*

$$\liminf_{N \to \infty} \frac{1}{N+1} \# \left\{ 0 \leq m \leq N : \sup_{s \in K} |\zeta(s + imh, \alpha) - f(s)| < \varepsilon \right\} > 0.$$

We propose the following extension of Theorem 1.

THEOREM 2. *Suppose that the set*

$$L(\alpha, h) = \left\{(\log(m + \alpha) : m \in \mathbb{N}_0), \tfrac{2\pi}{h}\right\}$$

is linearly independent over the field of rational numbers $\mathbb{Q}$. Let K and $f(s)$ be the same as in Theorem 1. Then the assertion of Theorem 1 is true.

For example, we can take $\alpha = \pi^{-1}$, $h = 2$.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

REMOVAL OF PARTICLES FROM GAS IN A MULTICHANNEL CYCLONE[1]

TERESĖ LEONAVIČIENĖ and PRANAS BALTRĖNAS

Vilnius Gediminas Technical University

Sauletekio av. 11, LT-10223 Vilnius, Lithuania

E-mail: `terese.leonaviciene@vgtu.lt, pranas.baltrenas@vgtu.lt`

Air pollution problems today are very important. Pollutants arise in many industrial processes. Air pollution causes some risks to the health and impact on life quality. The air quality questions are discussed at various levels. In order to improve the air quality the governments adopts legal acts and directives, air quality laws. To solve the air quality problems the industry can remove the particles from the air stream before it is emitted into the atmosphere.

Cyclone are widely used to separate dust and particles from gas streams. They are so popular due to their simple construction and economy [2]. Cyclone separators operate under the action of centrifugal forces. Neglect simple construction, it is difficult to analyze the particle motion in a cyclone. There are many parameters and they have influence to gas – particles flow in a cyclone.

We analyze the multichannel cyclone [1]. It consists of cylindrical and conical parts, inlet and outlet. The cylindrical part consists of four channels. Compare to usual cyclone, the cylindrical part of multichannel cyclone consists of four curvilinear channels. This construction leads to increase the separation efficiency. The gas stream with particles enters the cyclone through the tangential inlet. Due to the centrifugal forces, the particles are thrown towards the wall and fall into the hopper. Cleaned air outflows through the outlet.

Some assumptions and simplifications are used to determine the particle trajectories in the cylindrical part of the cyclone [3; 4]. The particles trajectories are calculated using a simple model based on force balance.

REFERENCES

[1] P. Baltrenas and P. Vaitiekunas and I. Jakstoniene and S. Konoverskyte. Study of Gas–Solid Flow in a Multichannel Cyclone. *Journal of Environmental Engineering and Landscape Management*, **20** (2): 129–137, 2012.

[2] M. Bohnet and O. Gottschalk and M. Morweiser. Modern design of aerocyclones. *Advanced Powder Technology*, **8** (2): 137–161, 1997.

[3] B. Wang and D. L. Xu and K. W. Chu and A. B. Yu. Numerical study of gas-solid flow in a cyclone separator. *Applied Mathematical Modelling*, 30: 1326–1342, 2006.

[4] H. Yoshida and K. Ono and K. Fukui. The effect of a new method of fluid flow control on submicron particle classification in gas-cyclones. *Powder Technology*, 149: 139–147, 2005.

[1]Research has been carried out within the framework of the project concerning the development of human resources A New Generation Multi-Channel Cyclone. Acronym DAKACIKAS (VP1-3.1-ŠMM-10-V-02-016)

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON MIXED JOINT UNIVERSALITY FOR SOME ZETA-FUNCTIONS

RENATA MACAITIENĖ

Institut of Informatics, Mathematics, E. studies, Šiauliai University

P. Višinskio str. 19, Šiauliai LT-77156, Lithuania

E-mail: renata.macaitiene@mi.su.lt

Since a remarkable Voronin's work [1] on the universality of the Riemann zeta-function, it is known that the majority of other zeta and L-functions also are universal in the sense that their shifts approximate uniformly on compact subsets of certain regions wide classes of analytic functions. Also, a more complicated approximation property of zeta and L-functions – the joint universality – is known. In this case, we deal with a simultaneous approximation of a given system of analytic functions. The first result in this direction also is due to Voronin who obtained [2] the joint universality of Dirichlet L-functions. H. Mishou began to study the joint universality for zeta-functions having and having no Euler product over primes. He proved [3] a joint universality theorem for the Riemann zeta-function and Hurwitz zeta-function (with transcendental parameter α).

In the talk, we also will consider so called mixed joint universality, i. e., we will show a new result [4] that every system of analytic functions can be approximated simultaneously uniformly on compact subsets of some region by a collection consisting of shifts of Dirichlet L-functions with pairwise non-equivalent characters and Lerch zeta-functions with specified parameters.

REFERENCES

[1] S. M. Voronin. Theorem on the universality of the Riemann zeta-function. *Izv. Akad. Nauk. SSSR, Ser. Matem.*, **39** (3): 475–486, 1975. (in Russian)

[2] S. M. Voronin. On the functional independence of Dirichlet L-functions. *Acta Arith.*, **27** : 493–503, 1975. (in Russian)

[3] H. Mishou. The joint value distribution of the Riemann zeta-function and Hurwitz zeta-functions. *Lith. Math. J.*, **47** (1): 32–47, 2007.

[4] A. Laurinčikas, R. Macaitienė. Joint Universality of Dirichlet L-Functions and Lerch Zeta-Functions. *Sibirskii Matematicheskii Zhurnal*, (submitted).

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

INVERSE SCATTERING PROBLEM FOR A PIECEWISE CONTINUOUS STURM-LIOUVILLE EQUATION WITH EIGENPARAMETER DEPENDENCE IN THE BOUNDARY CONDITION

KH.R. MAMEDOV and F.A. CETINKAYA

Department of Mathematics, Science and Letters Faculty, Mersin University

33343, Mersin, Turkey

E-mail: `hanlar@mersin.edu.tr, faycacetinkaya@mersin.edu.tr,`

In this study, the inverse problem of the scattering theory for a piecewise continuous Sturm-Liouville equation on the half line with boundary condition depending quadratic on the spectral parameter is considered. The scattering data of the problem are defined. Some properties of the scattering data are investigated. The fundamental equation is derived and uniqueness of algorithm to the potential with given scattering data is studied.

REFERENCES

[1] V. A. Marchenko. *Sturm-Liouville Operators and Their Applications.* AMS, Providence, 2011.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

NUMERICAL MODELLING AND OPTIMIZATION FOR THE MAGNETIC GEAR[1]

MAKSIMS MARINAKI

Faculty of Physics and Mathematics, University of Latvia

Zellu str. 8, Riga LV-1002, Latvia

Institute of Mathematics and Computer Science, University of Latvia

Raina bulvaris 29, Riga LV-1459, Latvia

E-mail: maksims.marinaki@lu.lv

Nowadays, magnetic gears are widely used in mechanical devices and machines. As a result, design optimization tasks gain an interest. We analyze an example: the mechanism involving rectangle-shaped magnets and its two dimensional model first proposed in [1].

We consider Maxwell's equations of magnetostatics in form (1):

$$\begin{cases} \nabla \times \mathbf{H} = 0, \\ \nabla \cdot \mathbf{B} = 0, \\ \mathbf{B} = \mu(\mathbf{H} + \mathbf{M}), \end{cases} \tag{1}$$

where $\mathbf{B}$ is the magnetic flux density, $\mathbf{H}$ is the magnetic field intensity, $\mathbf{M}$ is the magnetization, and μ is the material's magnetic permeability.

We pose a problem then by developing a 2D vector-potential formulation, and get the numerical finite element solution. Simulations are made in MATLAB and FEMM; results are shown in a way that we represent the calculated field $\mathbf{B}$ and get some insight for extrema of transmitted torque functional in different situations.

The task involving three parameters α, h and n is further considered. Those parameters respectively are angular displacement of the driver in magnetic gear, height of a permanent magnet and the number of magnetic pole pairs. PSO technique (particle swarm optimization) is applied for maximization of the functional $F(\mathbf{B}, \alpha, h, n)$, where F is the transmitted torque.
We obtain parameters that correspond to optimal design of the gear.

Another aspect is the reduction of computational time, that's why mesh moving methods are also discussed. Similarly to [2], we construct the auxiliary problems for each given set of parameters .

REFERENCES

[1] Yi-Chang Wu and Chih-Wen Wang. Transmitted Torque Analysis of a Magnetic Gear Mechanism with Rectangular Magnets. , Appl. Math. Inf. Sci, 2013.

[2] Shuangxia Niu, Ningning Chen, S. L. Ho, and W. N. Fu. Design Optimization of Magnetic Gears Using Mesh Adjustable Finite-Element Algorithm for Improved Torque. *IEEE TRANSACTIONS ON MAGNETICS*, **48** NO.11, 2012.

[1]This work was partially supported by the grant 623/2014 of the Latvian Council of Science

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ABOUT STABILITY AND CONVERGENCE OF FINITE-DIFFERENCE METHODS FOR IBVP FOR NON-LINEAR TIME-DEPENDENT PROBLEMS

PIOTR MATUS[1,2]

[1] *Institute of Mathematics, NAS of Belarus*

Surganov St. 11, 220072 Minsk, Belarus

[2] *Department of Mathematics The John Paul II Catholic University of Lublin*

Al. Raclawickie 14, 20-950 Lublin, Poland

E-mail: `matus@im.bas-net.by`

The present talk is devoted to the stability and convergence of finite-difference schemes for initial-boundary value problems for quasilinear parabolic equations with generalized solutions. The fundamental feature of the presented results is that they are obtained under the assumptions only on the input data[1; 2].

Sufficiently simple conditions on the input data are given to guarantee both the existence of the global smooth solution and the occurrence of blow-up in continuous and discrete cases for the multidimensional heat equation with the non-linear source. The presented mathematical apparatus is based on "student" technique of the method of energy inequalities[3].

A new mathematical apparatus for obtaining of two-sided estimates of the unknown solution on the basis of the proof of discrete analogues of Chaplygin and Hartman comparison theorems is presented [4]. In some sense it generalizes results of the discrete analogue of the Bihari lemma (the generalization of the Gronwall lemma on the non-linear case), which were obtained earlier by B.V. Demidovich.

An overview of the last results on the construction of the new classes of difference schemes for multidimensional quasi-linear convection-diffusion-reaction equations, which are exact on running waves, is given [5].

REFERENCES

[1] B. Jovanovic, M. Lapinska-Chrzczonowicz, A. Matus, and P. Matus. Stability of finite-difference schemes for ibvp for multidimensional parabolic equations with a nonlinear source of the power type. *Comp. Meth. Appl. Math.*, **12** (3):289–305, 2012.

[2] P. Matus. About convergence of difference schemes for IBVP for quasilinear parabolic equations with generalized solutions. *Comp. Meth. Appl. Math.*, **14** 2014. In print.

[3] P.P. Matus. On the Role of Conservation Laws in the Problem on the Onset of Unstable Solutions for Quasilinear Parabolic Equations and Their Approximations. *Differential Equations*, **49** (7):883–894, 2013.

[4] P.P. Matus, A. Paradzinska, and D.A. Schadinsky. Discrete analogs of the comparison theorems and their application. *Doklady of the NAS of Belarus*, **57** (4):16–20, 2013.

[5] M. Lapinska-Chrzczonowicz, P. Matus. Exact difference schemes for a two-dimensional convection-diffusion-reaction equation. *Computers & Mathematics with Applications*, 2014. http://dx.doi.org/10.1016/j.camwa.2014.01.022

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

CHARACTERISTICS OF CERTAIN PROCESS-DEPENDENT MATRICES

ZENONAS NAVICKAS and LIEPA BIKULCIENE and TADAS TELKSNYS

Kaunas University of Technology

Studentu 50, LT-51368, Kaunas, Lithuania

E-mail: tadas.telksnys@gmail.com

In light of recent advances regarding the use of algebraic data analysis methods in various fields of medicine, especially cardiology and complex system theory, a new mathematical theory concerned with analysing matrices of a certain type is being developed [1; 2; 3].

Given two time series x, y, second-order matrices having the following form are considered:

$$K_n^{(2)}(x,y) := \begin{bmatrix} x_n & x_{n+1} - y_{n+1} \\ x_{n-1} - y_{n-1} & y_n \end{bmatrix} \tag{1}$$

In the second-order case, scalar characteristics such as the trace, discriminant and other, more complex, eigenvalue-dependent coefficients are computed from matrices $K_n^{(2)}(x,y)$, where x and y are parameters of human vital signals. These parameters represent inter-relations between signals x and y and in the case of human vital signal type data, provide important information on the patients' physiological state.

$$K_n^{(3)}(x,y,z) := \begin{bmatrix} x_n & x_{n+1} - y_{n+1} & x_{n+1} - z_{n+1} \\ x_{n-1} - y_{n-1} & y_n & y_{n+1} - z_{n+1} \\ x_{n-1} - z_{n-1} & y_{n-1} - z_{n-1} & z_n \end{bmatrix} \tag{2}$$

In the third-order case, given three time series x, y, z, additive idempotent-nilpotent decompositions of (2) are formed. A finite matrix transformation sequence $T_1(\alpha_1), ..., T_k(\alpha_k), k \leq 6$ is used to fit the decomposition of a given matrix to a fixed form, generated by column-row products of a pair of inverse matrices (X_0, Y_0). The parameters of this fit $\alpha_1, ..., \alpha_k$ are considered as a type of transform of the three signals x, y, z and also represent inter-relations between the signals.

Examples of this type of analysis are provided using the approximations of solutions to Rössler's differential equations and human vital signal data registered during various medical tests. Human physiological state changes can be studied according to the dynamics of the characteristics mentioned above.

REFERENCES

[1] L. Bikulciene, Z. Navickas, R. Smidtaite, K. Berskiene, A. Vainoras. The measure of ECG complexity by matrix analysis. *Journal of Applied Electromagnetism*, **13** (1):1–9, 2011.

[2] A. Vainoras, R. Smidtaite, L. Bikulciene, L. Gargasas. Monitoring of elderly using special matrix characteristics for analysis of ECG parameters. In: *Biomedical engineering - 2011 : proceedings of international conference, Kaunas University of Technology, 27, 28 October 2011*, Kaunas University of Technology. Kaunas : Technologija, 2011, 26 – 29.

[3] R. Smidtaite, Z. Navickas, E. Venskaityte. ECG research using elements of matrix analysis and phase planes. *Electronics and Electrical Engineering*, **7** (103):83–86, 2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

STABILITY OF THE WEIGHTED FINITE-DIFFERENCE SCHEME FOR HYPERBOLIC EQUATION WITH TWO NONLOCAL INTEGRAL CONDITIONS

JURIJ NOVICKIJ and ARTŪRAS ŠTIKONAS

Dept. of Mathematics and Informatics, Vilnius University

Naugarduko 24, LT-03225, Vilnius, Lithuania

E-mail: jurij.novickij@gmail.com

Consider the hyperbolic equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = f(x,t), \qquad (x,t) \in \Omega \times (0,T], \tag{1}$$

where $\Omega = (0, L)$, with the classical initial conditions

$$u|_{t=0} = \phi(x), \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = \psi(x) \quad x \in \overline{\Omega} := [0, L], \tag{2}$$

and the additional nonlocal integral boundary conditions

$$u(0,t) = \gamma_0 \int_0^L u(x,t)\,dx + v_l(t), \quad u(1,t) = \gamma_1 \int_0^L u(x,t)\,dx + v_r(t), \quad t \in [0,T], \tag{3}$$

where $f(x,t)$, $\phi(x)$, $\psi(x)$, $v_l(t)$, and $v_r(t)$ are given functions, and γ_0 and γ_1 are given real parameters (see [1]). We are interested in sufficiently smooth solutions of the nonlocal problem (1)–(3).

PDE of the hyperbolic type with integral conditions often occur in problems related to fluid mechanics [2] (dynamics and elasticity), linear thermoelasticity [3], vibrations [4] etc.

We consider the stability of a weighted finite-difference scheme (with two weights σ_1 and σ_2) for a linear hyperbolic equation with nonlocal integral boundary condition. By studying the spectrum of the transition matrix of the three-layered difference scheme we obtain a sufficient stability condition in a special matrix norm.

REFERENCES

[1] F.F Ivanauskas, Yu. A. Novitski, M.P. Sapagovas, On the Stability of an Explicit Difference Scheme for Hyperbolic Equations with Nonlocal Boundary Conditions, *Differ. Equ.*, **49**(7):849–856, 2013.

[2] A.M. Nakhushev, An Approximate Method for Solving Boundary Value Problems for Differential Equations and Its Application to the Dynamics of Ground Moisture and Ground Water, *Differ. Uravn.*, **18**(1):72–81, 1982.

[3] A.F.Chudnovskii, *Teplofizika pochv* (Thermophysics of Soils), Moscow, 1976.

[4] V. F. Volkodavov, V. E. Zhukov, Two problems for the string vibration equation with integral conditions and special matching conditions on the characteristic, *Differ. Equ.*, **34**:501–505, 1998.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

COMONOTONE RATIONAL SPLINE HISTOPOLATION

PEETER OJA

Tartu University

J. Liivi 2, EE50409 Tartu, Estonia

E-mail: peeter.oja@ut.ee

We consider the histopolation problem with combined splines S of class C^1 being quadratic or linear/linear rational functions on each subinterval of a partition of the given interval $[a,b]$ as $a = x_0 < x_1 < ... < x_n = b$. We pose the histopolation conditions

$$\int_{x_{i-1}}^{x_i} S(x)dx = z_i(x_i - x_{i-1}), \quad i = 1, ..., n,$$

with given histogram heights z_i and additional boundary conditions. The existence, uniqueness and choice of kinds of intervals (quadratic or rational) are treated. The main problem in our talk is the convergence rate of such approximation procedure in the process $\max(x_i - x_{i-1}) \to 0$ when $z_i = (x_i - x_{i-1})^{-1} \int_{x_{i-1}}^{x_i} f(x)dx$ for a given sufficiently smooth function f.

REFERENCES

[1] M. Fischer, P. Oja. Monotonicity preserving rational spline histopolation. *Journal of Computational and Applied Mathematics*, **175** (2):195–208, 2005.

[2] M. Fischer, P. Oja. Convergence rate of rational spline histopolation. *Mathematical Modelling and Analysis*, **12** (1):1–10, 2007.

[3] M. Fischer, P. Oja, H. Trossmann. Comonotone shape-preserving spline histopolation. *Journal of Computational and Applied Mathematics*, **200** (1):127–139, 2007.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

PRICING OF THRESHOLD WARRANTS

ANDER OLVIK and RAUL KANGRO

Institute of Mathematical Statistics, University of Tartu

J. Liivi 2, 50409 Tartu, Estonia

E-mail: ander1@ut.ee, raul.kangro@ut.ee

Threshold warrants give right to buy specially issued stocks, if the performance of stock price satisfies some requirements. The combination of dilution effect [1] and exercising criterion that depends on actual transactions between market participants, changes the share price process of that company. Knowledge of stock price process is necessary for both pricing these warrants and any other already outstanding derivatives [2].

We use a model for firm price as a starting point for pricing such warrants. In our talk we demonstrate that the presence of threshold warrants either forces the stock to be untradeable for certain market scenarios or requires one to use incomplete and/or inefficient market models, where instead of replication argument, price of a derivative can be obtained by indifference pricing approach [3]. We assume that the stock is always tradeable, propose a possible stock price model and show numerical results of warrant pricing under different utility functions.

REFERENCES

[1] D. Galai, M.I. Schneller . Pricing of warrants and the value of the firm . *Journal of Finance* , **33** (5):1333-1342, 1978.

[2] M. Hanke, K. Pötzelberger . Consistent pricing of warrants and traded options . *Review of Financial Economics* , **11** (1):63-77, 2002.

[3] R. Carmona. *Indifference Pricing: Theory and Applications.* Princeton University Press, Princeton and Oxford, 2009.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

TWO-SAMPLE PROBLEMS OF SURVIVAL DATA

LEONORA PAHIRKO

Faculty of Physics and Mathematics, University of Latvia

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: leonorapahirko@inbox.lv

In survival analysis a sample of independent random variables, say $X_1,\dots,X_n$, usually may be interpreted as the collection of times from entry into a follow-up study until failure. Due to other causes of failure not all X_i's are observable, this situation is termed random censoring. Censored data often arise in biometry, reliability analysis and medical follow-up study.

In statistical analysis of survival data, it is often of interest to determine the difference between two treatment effects. Suppose that $X_1,\dots,X_n$ are control responses and $Y_1,\dots,Y_m$ are treatment responses with unknown distribution functions F_1 and F_2, respectively. Apart from graphical inspection we may be interested to check whether two samples differ by location and scale parameters, i.e., whether the relationship

$$F_1(t) = F_2\left(\frac{t-\mu}{\sigma}\right), \quad t \in \mathbb{R}$$

holds, where $\mu \in \mathbb{R}$ and σ is some positive constant. Valeinis ([1]) studied confidence bands for location-scale models via empirical likelihood method for uncensored data. We show that the limiting distribution differs in censored case due to parameter estimation.

The proposed methods for two-sample problems are based on Wang and Jing's ([2]) approach adjusting the empirical likelihood method for censored data in the one sample case.

REFERENCES

[1] J. Valeinis. *Confidence bands for structural relationship models.* Phd thesis, Goerg-August-University of Goettingen, Goettingen, 2007.

[2] Q. H. Wang and B. Y. Jing. Empirical likelihood for a class of functionals of survival distributions with censored data. *Annals of the Institute of Statistical Mathematics*, **53** (3):517–527, 2001.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

HIGH ORDER COMPACT FINITE DIFFERENCE SCHEME FOR EFFICIENT PRICING OF ASIAN OPTION BY PDE APPROACH

KULDIP SINGH PATEL and MANI MEHRA

Indian Institute of technology, Delhi

New Delhi, India

E-mail: kuldip@maths.iitd.ac.in, mmehra@maths.iitd.ac.in

Arithmetic average Asian option is studied in this paper. Partial differential equation(PDE) approach for pricing Asian option [3] is considered and the main difficulties encounters in standard finite difference schemes are explained. A numerical method is proposed for solving Asian call option PDE with a moving mesh [4]. The method uses high order compact finite difference scheme [1] for spatial discretization and the Rannacher time stepping scheme for the time discretization. It is observed that proposed numerical method provides accurate results as compared to finite difference method for large volatilities when other parameters are fixed. Moreover, results obtained here are also compared with the existing literature [4].

REFERENCES

[1] Sanjiva K.Lele. Compact finite difference schemes with spectral-like resolution. *Journal of computational physics*, **103** 16-42,1992.

[2] F.Dubois, T.Lelievre. Efficient pricing of Asian option by PDE approach. *Journal of computational Finance*, **8** (2)55-64,2004-2005.

[3] J. Vecer. A new PDE approach for pricing arithmatic average Asian option. *Journal of computational Finance*, **4** (4)105-113,2001.

[4] Zhongdi Cen, Anbo Le, and Aimin Xu. Finite difference scheme with a moving mesh for pricing Asian options. *Applied Mathematics and Computation*, **219** 8667-8675,2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ORDINARY AND GENERALIZED GREEN'S FUNCTIONS FOR DISCRETE NONLOCAL PROBLEMS

GAILĖ PAUKŠTAITĖ and ARTŪRAS ŠTIKONAS

Vilnius University

Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: gaile.paukstaite@mif.stud.vu.lt; arturas.stikonas@mif.vu.lt

We consider second order discrete boundary value problems with two nonlocal conditions

$$\mathcal{L}u := a_i^2 u_{i+2} + a_i^1 u_{i+1} + a_i^0 u_i = f_i, \quad i = 0, 1, 2, \ldots, n-2,$$
$$\langle L_j, u \rangle = 0, \quad j = 1, 2,$$

where nonlocal conditions are described by discrete linear functionals $L_j,\ j = 1, 2$.

According to [4], the problem has the unique solution if and only if the necessary and sufficient existence condition of ordinary discrete Green's function is satisfied. Even if this condition is not satisfied, every discrete problem always has a least squares solution. Generalized Green's function that describes the general least squares solution of discrete problem always exists as well and was investigated in [2].

In this paper, we compare the properties of generalized Green's function that describes the general least squares solution to properties of ordinary Green's function that describes the exact unique solution of nonlocal discrete problem. We can proof that generalized Green's function satisfies the same investigated properties of ordinary Green's function but in the sense of least squares.

Generalizations are also made for higher order discrete boundary value problems with nonlocal conditions.

REFERENCES

[1] A. Ben-Israel and T.N.E. Greville. *Generalized Inverses. Theory and Applications.* Springer Verlag, New York, 2003.

[2] G. Paukštaitė and A. Štikonas. Generalized Green's functions for the second order discrete problems with nonlocal conditions. *Lith. Math. J.*, **54** (2), 2014.

[3] R. Penrose. On best approximate solutions of linear matrix equations. *Proc. Cambridge Philos. Soc.*, **52** 17–19, 1956.

[4] S. Roman. *Green's functions for boundary-value problems with nonlocal boundary conditions.* Vilnius University, 2011. Doctoral dissertation

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

NUMERICAL SIMULATION OF THE MAGNETIC FLUID SEAL OF A ROTATING SHAFT

VIKTOR POLEVIKOV

Belarusian State University

Independence Ave. 4, 220030, Minsk, Belarus

E-mail: `polevikov@bsu.by`

Magnetic fluid seals are the best known technical application of magnetic fluids. Therefore, the development of an adequate mathematical model describing the relevant features of a magnetic fluid seal and numerical algorithms for its solution are of great importance, e.g. for the computer optimization of the design of magnetic fluid seals. The burst external pressure drop and the critical velocity of rotation of the shaft are the main operating characteristics of the seal. They are defined essentially by the shape and location of the sealing layer which is under the action of capillary, magnetic, centrifugal and pressure forces. An acceptable mathematical model of a magnetic fluid seal which takes into account the free surface deformation has been developed and realized in [1; 2]. In [1] the stability of a static magnetic fluid seal under the action of a pressure drop has been studied. In [2] the problem on magnetic fluid seal stability has been solved in a more general statement: the dynamic magnetic fluid seal has been considered under the action of both centrifugal forces and a pressure drop. Deficiency of the used models is the disregard of the diffusion process of ferromagnetic particles in the sealing layer under the influence of a nonuniform magnetic field.

As the next step toward the development of a realistic mathematical model we consider the influence of the magnetic particle diffusion process on the stability of the seal. In [3] the static magnetic fluid seal under an external pressure drop has been studied taking particle diffusion into consideration. The present study is devoted to the influence of the particle diffusion on the stability of the dynamic magnetic fluid seal of a rotating shaft. Mathematical model of the coupled problem consists of the magnetic field subproblem, the free-surface deformation subproblem, the hydrodynamic subproblem and the particle concentration subproblem. The subproblems are strictly interrelated to each other and are solved simultaneously. The non-inductive approximation which neglects magnetic field disturbances owing to the influence of magnetic fluid magnetization is used.

An algorithm for computer realization of the developed model is constructed and applied. Free surface shapes, azimuthal and secondary flows inside the sealing layer depending on the particle diffusion, and their influence on the stability of the seal are studied numerically. Numerical results are compared with those obtained in [2] in the approximation of a uniform particle concentration.

REFERENCES

[1] V. Polevikov. Stability of a static magnetic-fluid seal under the action of an external pressure drop. *Fluid Dynamics*, **32** (3):457–461, 1997.

[2] V. Polevikov and L. Tobiska. Modeling of a dynamic magneto-fluid seal in the presence of a pressure drop. *Fluid Dynamics*, **36** (6):890–898, 2001.

[3] V. Polevikov and L. Tobiska. Influence of diffusion of magnetic particles on stability of a static magnetic fluid seal under the action of external pressure drop. *Communications in Nonlinear Science and Numerical Simulation*, **16** :4021–4027, 2011.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

'NORTHEAST VOLATILITY WIND' EFFECT EVOLUTION RESEARCH BY USING NEURAL NETWORKS

ANDREJS PUCKOVS and ANDREJS MATVEJEVS

Riga Technical University

1 Kalku Street, Riga LV-1658, Latvia

E-mail: rostock@inbox.lv

The "North-East Volatility Wind" effect, desrib ed in [1] and [2] is selected for subsequent research. "North-East Volatility Wind" effect is described as effect of volatility transmission from low-frequncy components of the signal to high-frequency components. According to [1] in financial time series a "North-East Volatility Wind" effect is observed. This effect is traceable by using special approach approach applicable for stock indexes, allowing to reveal the instability of financial time series initially. This approach is based on time series (signal) decomposition into components by using wavelet filtering with subsequent volatility evolution research of each signal component. According to research, a slight increase in volatility in the low-frequency components of the signal leads to significant disturbances in high-frequency components destine entire signal volatility growth.

This approach is based on signal decomposition by using the wavelet filtering. Wavelet filtering is applied by using Direct and Inverse CWT for each scaling parameter. [3] Thus for each scaling parameter the signal component (which is part of the original signal) is obtained. For subsequent research volatility indicator is analysed by using 20-days time window, which is shifted on the time axis. Volatility analysis is done for each signal component. As a result volatility evolution in time is obtained for each signal component. As a next step volatility evolution crosscorrelation analysis is done for each signal component in order to describe volatility transmission from one volatility component to another. In other words volatility transmission from one volatility layer to another is analysed.

In current research volatility evolution crosscorrelation analysis is extended by using Neural Network Algorithms, to trace volatility evolution dependences in time, optimising Neural Network structure and finding correspondent weights. In current research complicated relationships between volatility layer are discovered.

As a result "North-East Volatility Wind" effect brings out deeper understanding of volatility evolution and opportunity to illuminate most dramatical market drawdowns initially. This opportunity is explained by ability to see a very small changes in volatility logarithm in the low-frequency components of the signal.

REFERENCES

[1] A. Puckovs and A. Matvejevs. Northeast Volatility Wind Effect. *Aplimat Journal*, **Vol.6,** 2014.ISSN 1337-6365

[2] A. Puckovs and A. Matvejevs. Equity Indexes Analysis and Synthesis by Using Wavelet Transform. In: *7th International Conference on Computational and Financial Econometrics (CFE 2013): Programme and Abstracts, United Kingdom, London, 13-16 December, 2013*, CFE-ERCIM 2013, Birkbeck, University of London, 2013, 98 – 98.

[3] A. Jakovlev. *Vvedenije v vejvlet preobrazovanija.* NGTU, Novosibirsk, 2003. (In Russian)

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

EVENTUALLY PERIODIC SOLUTIONS AND PATTERNS OF UNBOUNDED SOLUTIONS OF SECOND ORDER AND A DELAYED MAX-TYPE DIFFERENCE EQUATION.

MICHAEL A. RADIN

Rochester Institute of Technology

Rochester, New York U.S.A.

E-mail: marsma@rit.edu

We will begin our investigation of difference equations with eventually periodic solutions with the piece-wise linear difference equations; $3x + 1$ Conjecture and the Tent Map. We will then continue with the existence of eventually periodic solutions of the second order autonomous max-type difference equation. Afterwards, we will proceed existence of unbounded solutions of delayed non-autonomous max-type difference equations; in particular, the pattern of unbounded solutions. How many terms converge to 0 and how many terms diverge to infinity.

REFERENCES

[1] E.A. Grove, C. Kent, G. Ladas, and M. Radin, On $x_{n+1} = \max\left\{\frac{1}{x_n}, \frac{A_n}{x_{n-1}}\right\}$ with $\{A_n\}_{n=0}^{\infty}$ with a period-three parameter, Fields Inst. Commun. 29 (2001), pp. 161-180.

[2] C.M. Kent and M.A. Radin, On the boundedness nature of positive solutions of the difference equation $x_{n+1} = \max\left\{\frac{A_n}{x_n}, \frac{B_n}{x_{n-1}}\right\}$ with $\{A_n\}_{n=0}^{\infty}$ with periodic parameters, Proceedings of the Third International DCDIS Conference on Engineering Applications and Computational Algorithms 2003, Guelph, Ontario, Canada, May 15, 2003, pp. 11-15. Special Volume of the Dynamics of Continuous, Discrete and Impulsive Systems, Series B: Applications and Algorithms, Watam Press (2003).

[3] C. Kerbert and M. Radin, Unbounded solutions of the max-type difference equation $x_{n+1} = \max\left\{\frac{A_n}{x_n}, \frac{B_n}{x_{n-2}}\right\}$, Central European Journal of Mathematics 6(2) (2008), pp. 307-324.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

MODELING AND SIMULATIONS OF BEAM QUALITY IMPROVEMENT IN BROAD AREA SEMICONDUCTOR DEVICES

MINDAUGAS RADZIUNAS

Weierstrass Institute for Applied Analysis and Stochastics

Mohrenstrasse 39, 10117 Berlin, Germany

E-mail: Mindaugas.Radziunas@wias-berlin.de

High power high brightness edge-emitting broad area semiconductor (BAS) lasers and optical amplifiers are compact devices playing a key role in different laser technologies. These devices have a relatively simple geometry allowing an efficient electrical pumping and are able to operate at the high power (tens of Watts) regimes. BAS devices, however, have one serious drawback: once operated at high power regimes, they suffer from a relatively low beam quality which is due to simultaneous irregular contribution of different lateral and longitudinal modes. As a result, the emitted optical beam has undesirable broad optical and lateral spectra. A quality improvement of the beam amplified in BAS amplifiers or generated by BAS lasers is a very important issue of the modern semiconductor laser technology, and there are several BAS device concepts allowing some improvement of the optical beam.

In this talk some of these schemes [1; 2] allowing an improvement of the beam quality in the BAS devices will be discussed. Our theoretical study is based on the simulations of the following 2+1-dimensional traveling wave model [3]:

$$\begin{aligned}
&\tfrac{\partial}{\partial x}E^{\pm} \pm \tfrac{\partial}{\partial z}E^{\pm} = -\tfrac{i}{2}\tfrac{\partial^2}{\partial x^2}E^{\pm} - \left[i\beta(N,|E|^2)+\mathcal{D}\right]E^{\pm} - i\kappa E^{\mp} + F_{sp}^{pm},\\
&\mathcal{D}E^{\pm} = \tfrac{\bar{g}}{2}\left(E^{\pm}-P^{\pm}\right), \qquad \tfrac{\partial}{\partial t}P^{\pm} = i\bar{\omega}P^{\pm} + \bar{\gamma}\left(E^{\pm}-P^{\pm}\right),\\
&\tfrac{1}{\mu}\tfrac{\partial}{\partial t}N = D\tfrac{\partial^2}{\partial x^2} + I(z,x) - R(N) - \Re e\sum_{\nu=\pm} E^{\nu *}\left[G(N,|E|^2)-2\mathcal{D}\right]E^{\nu}
\end{aligned}$$

and solved in the domain $(t,x,z) \in [0,T]\times\mathbf{R}\times[0,L]$. Here, slowly varying complex amplitudes of the counter-propagating optical fields $E^{\pm}$ satisfy the reflectivity boundary conditions at the longitudinal borders of the domain, $z=0$ and $z=L$. A proper resolution of the model equations in laterally truncated domain requires a fine space (up to 10^7 mesh points) and time discretization. The resulting large numerical scheme is solved using multilevel parallel distributed computing, that allows us to run long time dynamic simulations over large parameter ranges in reasonable time [4].

REFERENCES

[1] M. Radziunas and K. Staliunas. Spatial rocking in broad area semiconductor lasers. *Europhysics Letters*, **95** :14002, 2011.

[2] M. Radziunas et al.. Intrinsic beam shaping mechanism in spatially modulated broad area semiconductor amplifiers. *Appl. Phys. Lett.*, **103** (13):132101, 2013.

[3] M. Spreemann et al.. Measurement and simulation of distributed-feedback tapered Master-Oscillators Power-Amplifiers. *IEEE J. of Quantum Electron.*, **45** :609–616, 2009.

[4] M. Radziunas and R. Čiegis. Modeling and simulations of beam stabilization in edge-emitting broad area semiconductor devices. In: *Lecture Notes in Computer Science*, Proceedings of the 10th International Conference on Parallel Processing and Applied Mathematics (PPAM 2013), Springer, 2014.(to appear)

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

INVESTIGATIONS OF THE ASYMPTOTIC BEHAVIOUR OF PERIODIC HURWITZ ZETA-FUNCTIONS

AUDRONĖ RIMKEVIČIENĖ

Department of Electrical Engineering, Šiauliai State College

Aušros al. 40, LT-76241, Šiauliai, Lithuania

E-mail: `audronerim@gmail.com`

Let $\mathfrak{a} = \{a_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers, and α, $0 < \alpha \leq 1$, be a fixed parameter. In the report, we discuss a characterization by limit theorems of the asymptotic behaviour of the periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$, $s = \sigma + it$, which is defined, for $\sigma > 1$, by the

$$\zeta(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m + \alpha)^s}.$$

and by analytic continuation elsewhere. We will present one-dimensional and multi-dimensional limit theorems on the weakly convergent probability measures on the complex plane which are defined by terms of periodic Hurwitz zeta-functions. The limit measures in these theorems are explicitly given, and depend on the arithmetical nature of the parameter α. In one-dimensional theorems, the cases of transcendental, rational and algebraic irrational α are considered. In multi-dimensional limit theorems, we dent with algebraically independent, rational, and algebraic irrational satisfying a certain independence condition parameters. In the case of algebraic irrational parameters, a new mapping hypothesis is also applied. The results obtained constitute the doctoral dissertation prepared externally by the author.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

BLOCK-WISE GEOMETRIC MULTIGRID FOR CELL-CENTERED DISCRETIZATIONS IN SEMI-STRUCTURED TRIANGULAR GRIDS

CARMEN RODRIGO

IUMA & Department of Applied Mathematics, University of Zaragoza

María de Luna, 3, 50018 Zaragoza, Spain

E-mail: carmenr@unizar.es

In this work we propose a block-wise multigrid algorithm on semi-structured triangular grids for solving piecewise constant diffusivity problems on relatively complex domains. Cell-centered finite volume schemes for this kind of problems are considered. Due to the semi-structured character of the mesh, on each structured patch, different smoothers can be considered depending on the geometry of the grid. In this way, the multigrid method is constructed in a block-wise form, and its global behavior will rely on the components on each block. To make an appropriate choice on each structured patch, a special local Fourier analysis for cell-centered discretizations on triangular grids has been performed. Simple inter-grid transfer operators are chosen to facilitate the communication between two consecutive patches, and the difficulties appearing when highly varying coefficients occur are overcome by the use of a modified Galerkin coarse grid approximation. Numerical experiments are presented to illustrate the good behavior of the proposed multigrid method which achieves an h-independent convergence rate.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ON THE ROLE OF THE MITTAG-LEFFLER FUNCTION IN FRACTIONAL MODELLING

SERGEI ROGOSIN, MARINA DUBATOVSKAYA

Belarusian State University

4, Nezavisimosti ave, BY-220030 Minsk, Belarus

E-mail: rogosin@bsu.by

The survey report illustrated the distinguished role of the Mittag-Leffler function in the fractional modelling and solution of the corresponding fractional problems is presented. This function, introduced by G.M. Mittag-Leffler at the beginning of XX century in form of series

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}, \quad \alpha > 0, \tag{1}$$

appeared to be very important generalization of the exponential function [5; 6].

First mathematical application was proposed in [2], where the solution of the second kind of the Abel integral equation was given in terms of Mittag-Leffler function. Later many genarilazations of this function were considered (see, e.g., [1; 3]), but the role of the Mittag-Leffler is mainly due to its relation to Fractional Calculus (see [6; 7]).

Among the applications of the Mittag-Leffler function we can single out

- solution of fractional integral and differential equations in terms of this function and its generalization (see, e.g., [1; 3]);
- solution of fractional deterministic problems (such as fractional relaxation and oscillation, problems of fractional visco-elasticity etc., see, e.g., [1; 4; 7]);
- solution of fractional stochastic problems (involving fractional Poisson processes, renewal processes of Mittag-Leffler type, fractional diffusion processes etc., see, e.g., [1; 4; 7]).

In the report, it will be described some properties of Mittag-Leffler function and its generalizations and their role in fractional modelling. Concrete mathematical problems will be presented and some applied models (from physics, mechanics, chemistry etc.) which are formulated and discussed with the help of these functions will be considered.

REFERENCES

[1] R. Gorenflo, A. Kilbas, F. Mainardi and S. Rogosin. *Mittag-Leffler functions, related topics and applications.* Springer Verlag, (submitted).

[2] E. Hille, J.D. Tamarkin. On the theory of linear integral equations. *Ann. Math.*, **31** . 479–528, 1930.

[3] A.A. Kilbas, H.M. Srivastava and J.J. Trujillo. *Theory and Applications of Fractional Differential Equations.* Elsevier, Amsterdam etc.. 2006

[4] F. Mainardi. *Fractional calculus and waves in linear viscoelasticity.* Imperial College Press, London. 2010

[5] A.M. Mathai, H.J. Haubold. *Special Functions for Applied Scientists.* Springer Verlag, New York. 2008

[6] S.G. Samko, A.A. Kilbas and O.I. Marichev. *Fractional Integrals and Derivatives: Theory and Applications.* Gordon and Breach, New York and London. 1993

[7] V.V. Uchaikin. *Fractional derivatives for physicists and engineers, Vol. I and II.* Springer Verlag/Higher Education Press, Berlin/Beijing. 2013

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ECONOMETRIC MODELLING OF THE IMPACT OF GLOBAL AND MACROECONOMIC PROCESSES ON THE STOCK MARKET PERFORMANCE: THE CASE OF THE BALTIC COUNTRIES

RIMANTAS RUDZKIS[1] AND ROMA VALKAVIČIENĖ[2]

[1] *Institute of Informatics and Mathematics*

Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: rimantas.rudzkis@mii.vu.lt

[2] *Vilnius Gediminas Technical University*

Saulėtekio av. 11, LT-10223 Vilnius, Lithuania

E-mail: roma.uzdanaviciute@gmail.com

There are a large number of empirical investigations on the relationship between global, macroeconomic variables and stock prices in the developed economies such as USA, Great Britain, Japan, Italy, Spain, France and others. However, the economic role of stock markets in relatively small open dynamically developing countries is less clear. Specifically, how do less developed markets respond to changes in their fundamental economic variables, compared with the well-developed and more efficient markets? The Baltic countries is that example of small open emerging economy in such cases: their economy has become much more related to the other economies, as international trade has expanded and financial integration with the rest of the world has deepened. There is evidence that existing links, for example, share prices of stock markets, may have become more important in the last few years. In this talk we address the issue of the econometric modelling of the Baltic countries economy in order to investigate the relative performance of rules that systematically respond to share prices and those that do not. Due to the sheer number of factors and shortness of the existing time series, statistical model development of OMX Baltic securities market involves several stages. It is extremely important to determine the most informative factors – their determination on the created models is defined by an exploratory analysis. Also the research methodology is backed with the findings of Lithuanian and foreign scientists from an extensive overview of specific literature. This study concludes that global and macroeconomic variables are significant in predicting changes in stock prices of the Baltic countries and can vary considerably depending on the individual sector's price indices. As well as the research provides investors who are shaping their portfolios taking into account the macroeconomic forecasts with additional opportunities on the basis of sectoral stock price indices regression equations.

REFERENCES

[1] R. Rudzkis, R. Uzdanavičiūtė. Econometric models of the impact of macroeconomic processes on the stock market in the Baltic countries. *Technological and Economic Development of Economy*, (per review).

[2] A. Humpe, P. Macmillan. Can macroeconomic variables explain long–term stock market movements? A comparison of the US and Japan. *Applied Financial Economics* , **19 (2)** 111–119,2009.

[3] C.A. Yartey. The institutional and macroeconomic determinants of stock market development in emerging economies. *Applied Financial Economics*, 20 (21), 1615–1625, 2010.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

THE EXISTENCE RESULTS FOR SOME NONLINEAR BOUNDARY VALUE PROBLEM

NATALIJA SERGEJEVA

Institute of Mathematics and Computer Science, University of Latvia

Rainis blvd. 29, Riga LV-1459, Latvia

E-mail: natalijasergejeva@inbox.lv

The existence results are established for the nonlocal boundary value problem

$$\begin{aligned} &x'' = -\mu x^+ + \lambda x^- + h(t, x, x'),\\ &x'(0) = 0, \quad \int_0^1 x(s)ds = 0, \end{aligned}$$

provided that $h(t, x, x')$ is continuous and Lipschitzian in x and x'.

The results are based on the study of spectrum for the problem

$$x'' = -\mu x^+ + \lambda x^-, \quad x'(0) = 0, \quad \int_0^1 x(s)ds = 0.$$

REFERENCES

[1] A. Kufner, S. Fučík. *Nonlinear Differential Equations.* Elsevier, Amsterdam, 1980.

[2] N. Sergejeva. On the Fučík type problem with the integral and Neumann conditions. *Proc. of IMCS of University of Latvia*, **13** 98–103, 2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

GUARANTEED ACCURACY OF THE SOLUTION TO THE STANDARD DIFFERENCE SCHEME IN THE PRESENCE OF PERTURBATIONS FOR A SINGULARLY PERTURBED CONVECTION-DIFFUSION EQUATION

GRIGORII SHISHKIN

Institute of Mathematics and Mechanics, Ural Branch of Russian Academy of Sciences

S. Kovalevskaya Str., 16, 620990, Yekaterinburg, Russia

E-mail: `shishkin@imm.uran.ru`

At present, numerical methods based on standard finite difference schemes are well developed and widely used for solving scientific problems, as well as in applications [1]. Quite often such methods are used to solve problems with boundary layers. For convergence of a solution to the standard difference scheme, it is sufficient to use a uniform grid under the condition that its grid step-size is much less than the value of the perturbation parameter ε [2; 3]. For advanced supercomputers such a condition does not seem restrictive.

However, as shown by recent studies, in the case of a singularly perturbed convection-diffusion equation, standard schemes on uniform grids are not ε-uniformly well-conditioned, and, as a consequence, ε-uniformly stable to perturbations in the data of the grid problem, or to the computer perturbations [4; 5]. For small values of the parameter ε, errors caused by perturbations can be much larger than the theoretical errors in the absence of perturbations (or, in short, theoretical errors), leading to incorrect results. Identifying these incorrect results, in general, it is difficult, because perturbations in the data and computer perturbations are probabilistic in nature.

Depending on the relation between theoretical errors and errors caused by the perturbations, the results obtained can be guaranteed (errors of the perturbations do not exceed theoretical errors in order), adequate (errors of the perturbations are of the same order as the theoretical errors) or noisy (errors of the perturbations exceed the theoretical errors in order). In the present talk, we discuss conditions imposed on the perturbations under which accuracy of the solution to the standard difference scheme in the presence of perturbations is guaranteed; results of numerical experiments are given.

This research was supported by the Russian Foundation for Basic Research under grant No.13-01-00618.

REFERENCES

[1] A. Samarskii. *Theory of Difference Schemes.* Marcel Dekker, New York, 2001.

[2] P. Farrell, A. Hegarty, J. Miller, E. O'Riordan and G. Shishkin. *Robust computational techniques for boundary layers.* CRC Press, Boca Raton, 2000.

[3] G. Shishkin and L. Shishkina. *Difference Methods for Singular Perturbation Problems.* CRC Press, Boca Raton, 2009.

[4] G. Shishkin. Conditioning of a difference scheme of the solution decomposition method for a singularly perturbed convection-diffusion equation. *Trudy IMM UrO RAN*, **18** (2):291–304, 2012.in Russian

[5] G. Shishkin. Stability of a Standard Finite Difference Scheme for a Singularly Perturbed Convection-Diffusion Equation. *Doklady Mathematics*, **87** (1):107–109, 2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

SCHEME OF IMPROVED ACCURACY ORDER BASED ON THE SOLUTION DECOMPOSITION METHOD FOR A SINGULARLY PERTURBED REACTION-DIFFUSION EQUATION

LIDIA SHISHKINA

Institute of Mathematics and Mechanics, Ural Branch of Russian Academy of Sciences

S. Kovalevskaya Str., 16, 620990, Yekaterinburg, Russia

E-mail: Lida@convex.ru

Recently, for singularly perturbed problems, ε-uniformly ($\varepsilon \in (0,1]$) convergent difference schemes of improved accuracy order were constructed on the basis of a new approach using schemes of the solution decomposition method and Richardson extrapolation technique. Unlike the known approach that uses special piecewise-uniform meshes, condensing in the boundary layer (see, e.g., [1] and the bibliography therein), in the new approach, grid subproblems are solved on the corresponding uniform grids, moreover, coefficients in grid equations do not depend on the explicit form of the singular components in the solution. In [2] for a Dirichlet problem for a singularly perturbed parabolic reaction-diffusion equation, an improved scheme based on this approach was constructed that converges at the rate $\mathcal{O}\left(N^{-4}\ln^4 N + N_0^{-2}\right)$, where $N+1$ and N_0+1 are the number of nodes in the spatial and time meshes, respectively. On the basis of the same approach, in [3] for a Dirichlet problem for a singularly perturbed ordinary differential reaction-diffusion equation, an improved scheme of the solution decomposition method was constructed approximating ε-uniformly both the solution of the differential problem and its normalized derivatives up to the second order that converges at the rate $\mathcal{O}\left(N^{-4}\ln^4 N\right)$, where $N+1$ is the number of grid nodes used.

In the present research, for a Dirichlet problem for a singularly perturbed ordinary differential reaction-diffusion equation, the Richardson difference scheme of the solution decomposition method was constructed in the case of two embedded grids that converges ε-uniformly at the rate $\mathcal{O}\left(N^{-4}\ln^4 N\right)$. The proposed new version of the difference scheme allows to construct improved difference schemes for the number of embedded meshes more than two (under the construction of schemes that converge with order close to six and higher) and, unlike the papers [2], and [3], it does not require the use of grid constructions whose definition domain is beyond the definition domain of the boundary value problem.

This research was supported by the Russian Foundation for Basic Research under grant No.13-01-00618.

REFERENCES

[1] G. Shishkin and L. Shishkina. *Difference Methods for Singular Perturbation Problems.* CRC Press, 2009.

[2] G. Shishkin and L. Shishkina. A Richardson Scheme of the Decomposition Method for Solving Singularly Perturbed Parabolic Reaction-Diffusion Equation. *Comp. Math. Math. Phys.*, **50** (12):2003–2022, 2010.

[3] G. Shishkin and L. Shishkina. Improved Approximations of the Solution and Derivatives to a Singularly Perturbed Reaction-Diffusion Equation Based on the Solution Decomposition Method. *Comp. Math. Math. Phys.*, **51** (06):1020–1049, 2011.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ZERO-DISTRIBUTION OF SOME FUNCTIONS RELATED TO PERIODIC ZETA-FUNCTIONS

DARIUS ŠIAUČIŪNAS

Institute of Informatics, Mathematics and e-studies, Šiauliai University

P. Višinskio str. 19, LT-77156 Šiauliai, Lithuania

E-mail: siauciunas@fm.su.lt

Let $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ and $\mathfrak{b} = \{b_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ be the two periodic sequences of complex numbers. In the report, we consider the zero-distribution of some functions related to the periodic zeta-function $\zeta(s;\mathfrak{a})$ and periodic Hurwitz zeta-function $\zeta(s,\alpha;\mathfrak{b})$ with parameter α, $0 < \alpha \le 1$, which are defined, for $\sigma > 1$, by the Dirichlet series

$$\zeta(s;\mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s} \qquad \text{and} \qquad \zeta(a,\alpha,\mathfrak{b}) = \sum_{m=0}^{\infty} \frac{b_m}{(m+\alpha)^s},$$

and by analytic continuation elsewhere.

We give one example of functions which will be discussed in our report. Let

$$\underline{\zeta}(s,\alpha;\mathfrak{a},\mathfrak{b}) = c_1\zeta(s;\mathfrak{a}) + c_2\zeta(s,\alpha;\mathfrak{b}), \quad c_1, c_2 \in \mathbb{C} \setminus \{0\}.$$

THEOREM 1. *Suppose that the number α is transcendental, the sequence $\mathfrak{a}$ is multiplicative, and, for each prime p, the inequality*

$$\sum_{m=1}^{\infty} \frac{|a_{p^m}|}{p^{\frac{m}{2}}} \le c < 1$$

is satisfied. Then, for any σ_1, σ_2, $\frac{1}{2} < \sigma_1, \sigma_2 < 1$, there exists a constant $c = c(\sigma_1, \sigma_2, \alpha, \mathfrak{a}, \mathfrak{b}) > 0$ such that, for sufficiently large T, the function $\underline{\zeta}(s,\alpha;\mathfrak{a},\mathfrak{b})$ has more than cT zeros in the rectangle

$$\sigma_1 < \sigma < \sigma_2, \qquad 0 < t < T.$$

A similar result is also true for more general functions $F(\zeta(s;\mathfrak{a}), \zeta(s,\alpha;\mathfrak{b}))$, where the operator $F : H^2(D) \to H(D)$, $H(D)$ is the space of analytic functions on $D = \left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$, satisfies a modification of the Lipschitz condition.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

INVESTIGATION OF CRITICAL AND BIFURCATION POINTS FOR STURM–LIOUVILLE PROBLEM WITH INTEGRAL BOUNDARY CONDITION

AGNĖ SKUČAITĖ

Vilnius University, Institute of Mathematics and Informatics

Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: `agne.skucaite@mii.vu.lt`

We investigate Sturm–Liouville problem with one classical boundary condition

$$-u'' = \lambda u, \quad x \in (0,1), \qquad u(0) = 0, \tag{1}$$

and another nonlocal boundary condition (NBC):

$$u(1) = \gamma \int_{\xi_0}^{\xi_1} u(x)\,\mathrm{d}x \quad \text{or} \quad u(1) = \gamma \int_0^{\xi} u(x)\,\mathrm{d}x \tag{2}$$

with parameters $\gamma \in \mathbb{R}$ and $0 < \xi_0 < \xi_1 < 1$ or $0 < \xi < 1$ [2; 3].

New results on critical point of characteristic function of problems (1)–(2) are given.

In the paper [1] the similar problem is investigated for operator with two-point boundary condition. The eigenvalue problems, investigation of the spectra, analysis of nonnegative solutions for the operators with NBCs of Bitsadze–Samarskii or of integral-type are given in the papers [1; 2]. Complex eigenvalues for differential operators with NBCs are less investigated than the real case.

We investigate how the bifurcation points depend on some boundary condition parameters and analyze how complex eigenvalues of these problems depend on the parameters of nonlocal integral boundary conditions. We present the results of modelling and computational analysis and illustrate the existing situation in graphs.

REFERENCES

[1] A. Štikonas and O. Štikonienė. Characteristic Functions for Sturm-Liouville Problems with Nonlocal Boundary Conditions. *Math. Model. Anal.*, **14** (2):229–246, 2009.

[2] A. Skučaitė, K. Skučaitė-Bingelė, S. Pečiulytė and A. Štikonas. Investigation of the Spectrum for the Sturm-Liouville Problem with one Integral Boundary Condition. *Nonlinear Anal. Model. Control*, **15** (4):501–512, 2010.

[3] S. Mikalauskaitė. *Investigation of the Spectrum of the Sturm–Liouville Problem with Nonlocal Condition Depending on Three Parameters.* Vytautas Magnus University, May 2011. Kaunas,(in Lithuanian), Master's Thesis.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

INVESTIGATION OF CRITICAL POINTS FOR STURM–LIOUVILLE PROBLEM WITH TWO-POINT NONLOCAL BOUNDARY CONDITION

K. SKUČAITĖ-BINGELĖ

Vilnius University, Institute of Mathematics and Informatics

Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: `kristina.skucaite-bingele@mii.vu.lt`

Let us investigate the Sturm–Liouville problem

$$-u'' = \lambda u, \quad t \in (0,1), \tag{1}$$

with one classical (2) and the second two-point Bicadze–Samarski type (3) nonlocal boundary condition

$$u(0) = 0, \tag{2}$$
$$u(1) = \gamma u'(\xi), \tag{3}$$

with parameters $\gamma \in \mathbb{R}$ and $\xi \in (0,1)$.

Characteristic function for Sturm–Liouville problem with one classical and other nonlocal two-point boundary conditions is analysed in the papers [3; 4]. In these papers investigated constant eigenvalue point, complex and real characteristic functions. The critical points for such type nonlocal boundary problems are less investigated. In the papers [1; 2] the similar problems critical points is investigated for nonlocal two-point or integral boundary conditions.

We analyze Sturm–Liouville problem and investigate how distribution of the critical points of spectrum of this problem depends on the parameters γ and ξ of the nonlocal boundary conditions. Many results are presented as a graphs of the critical point and characteristic functions.

REFERENCES

[1] S. Pečiulytė, A. Štikonas. Distribution of the Critical and other Points in Boundary Problems with Nonlocal Boundary Condition. *Lith. Mathem. J.*, **47** spec. no.:405–410, 2007.

[2] S. Pečiulytė, O. Štikonienė, A. Štikonas. Investigation of Negative Critical Points of the Characteristic Functions for Problems with Nonlocal Boundary Condition. *Nonlin. Anal.: Modell. and Contr.*, **13** (4):467–490, 2008.

[3] K. Skučaitė, S. Pečiulytė, A. Štikonas. Investigation of Complex Eigenvalues for Stationary Problems with two-point Nonlocal Boundary Conditions. *Mathem. and Mathemat. Modell.*, (5):24–32, 2009.

[4] A. Štikonas, O. Štikonienė. Characteristic Functions of Sturm–Liouville Problems with Nonlocal Boundary Condition. *Mathemat. Modell. and Anal.*, **14** (2):229–246, 2009.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

NONLOCAL BOUNDARY VALUE PROBLEM FOR A DIFFERENTIAL EQUATION ARISING IN BOUNDARY LAYER THEORY[1]

SERGEY SMIRNOV

Institute of Mathematics and Computer Science, University of Latvia

Raina bulvaris 29, Riga, LV-1459, Latvia

E-mail: `srgsm@inbox.lv`

The steady motion in the boundary layer along a thin flat plate which is immersed at zero incidence in a uniform stream with constant velocity can be described in terms of the solution of the Blasius differential equation

$$x''' = -xx'',$$

which satisfies the boundary conditions

$$x(0) = x'(0) = 0,\ x'(\infty) = 1.$$

It is our goal to study the existence of solutions to the generalized boundary value problem consisting of the nonlinear third order differential equation

$$x''' = -a(t)f(x)x'' \tag{1}$$

subject to the nonlocal boundary conditions

$$x(0) = x'(0) = 0,\ x'(\infty) = x(\xi), \tag{2}$$

where $0 < \xi < +\infty$.

REFERENCES

[1] H. Blasius. Grenzschichten in Flussigkeiten mit kleiner Reibung. *Z. Math. Phys.*, **56** : 1–37, 1908.

[2] W.A. Coppel. On a Differential Equation of Boundary-Layer Theory. *Phil. Trans. R. Soc. Lond.*, **253** : 101–136, 1960.

[3] Stuart P. Hastings and J. Bryce McLeod. *Classical Methods in Ordinary Differential Equations: With Applications to Boundary Value Problems.* American Mathematical Society, 2012.

[4] S. Pečiulytė and A. Štikonas. Sturm-Liouville Problem for Stationary Differential Operator with Nonlocal Two-Point Boundary Conditions. *Nonlinear Analysis: Modelling and Control*, **11** : 47–78, 2006.

[5] H. Schlichting and K. Gersten. *Boundary Layer Theory.* Springer-Verlag, Berlin, Heidelberg, 2000.

[1]This research has been supported by the European Social Fund within the project Nr. 2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

A SURVEY ON STATIONARY PROBLEMS, GREEN'S FUNCTIONS AND SPECTRUM OF STURM–LIOUVILLE PROBLEM WITH NONLOCAL BOUNDARY CONDITIONS

ARTŪRAS ŠTIKONAS

Vilnius University

Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: `arturas.stikonas@mif.vu.lt`

Boundary Value Problems (BVP) involving Ordinary Diferential Equations (ODE) arise in mathematics, applied mathematics, in physical science. In mathematics the main questions are existence and uniqueness of such problems. The new class of BVP are problems with Nonlocal Boundary Conditions (NBC). NBCs depend on parameters. Although at the present moment mostly nonlinear (quasi-linear) problems are studied, but investigation of linear differential operators with nonconstant coefficients and linear NBCs are actual up-to-date problem. We present a survey of recent results on the existence of solutions of Stationary Problems with NBC and we will summarised basic results in literature related to:

- existence and uniqueness domain in the space of parameters;
- Green's functions;
- properties of the Spectrum for Sturm–Liouville problem with NBCs;
- characteristic functions for BVP with NBC;
- properties of a characteristic curve;
- applications to non-stationary problems;
- investigation of Finite Difference Schemes for various stationary and non-stationary problems;
- results of the Lithuanian scientists.

REFERENCES

[1] E.A. Coddington and N. Levison. *Theory of Ordinary Differential Equations.* McGraw Hill Book Co., Inc., New York, Toronto, London, 1955.

[2] D.G. Duffy. *Green's Functions with Applications.* Chapman & Hall/CRC Press, 2001.

[3] M. Dehghan. Efficient techniques for the second-order parabolic equation subject to nonlocal specifications. *Applied Numerical Mathematic*, **52** (1):39–62, 2005.

[4] W. Ge. *Boundary Value Problems for Ordinary Differential Equations.* Science Press, Beijing, 2007.

[5] R. Ma. A Survey On Nonlocal Boundary Values Problem. *Appl. Math. E-notes*, **7** :257–279, 2007.

[6] S.K. Ntouyas. Nonlocal initial and boundary value problems: A survey. In: *Handbook of Differential Equations. Ordinary Differential Equations*, Eds. A. Cañada and P. Drábek and A. Fonda, North-Holland, Elsevier, Vol. 2, 461–558, 2005.

[7] A. Štikonas and O. Štikonienė. Characteristic Functions for Sturm–Liouville Problems with Nonlocal Boundary Conditions. *Math. Model. Anal.*, **14** (2):229–246, 2009.

[8] A. Štikonas. Investigation of Characteristic Curve for Sturm–Liouville Problem with Nonlocal Boundary Conditions on Torus. *Math. Model. Anal.*, **16** (1):1–22, 2011.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ALTERNATING DIRECTION PEACEMAN-RACHFORD METHOD FOR PSEUDOPARABOLIC EQUATION WITH NONLOCAL CONDITIONS

OLGA ŠTIKONIENĖ[1,2] and MIFODIJUS SAPAGOVAS[1]

[1] *Vilnius University, Institute of Mathematics and Informatics,* [2] *Vilnius University, Department of Mathematics and Informatics*

[1]Akademijos 4, LT-08663, Vilnius, Lithuania, [2]Naugarduko 24, LT-03225, Vilnius, Lithuania

E-mail: olga.stikoniene@mif.vu.lt, mifodijus.sapagovas@mii.vu.lt

We consider the third order linear pseudoparabolic equation with nonlocal integral conditions

$$u_t = u_{xx} + u_{yy} + \eta(u_{xx} + u_{yy})_t + f(x,y,t), \quad x, y \in D = \{x \in (0, L_1), y \in (0, L_2), \tag{1}$$

$$u(0,y,t) = \gamma_1 \int_0^{L_1} u(x,y,t)\mathrm{d}x + \mu_1(t), \tag{2}$$

$$u(L_1,y,t) = \gamma_2 \int_0^{L_1} u(x,y,t)\mathrm{d}x + \mu_2(t), \tag{3}$$

$$u(x,0,t) = \mu_3(x,t), \qquad u(x,L_2,t) = \mu_4(x,t), \tag{4}$$

where $\eta \geq 0$. Let us consider three-layer finite difference scheme (FDS) for problem (1)–(4):

$$\frac{u_{ij}^{n+1/2} - u_{ij}^n}{\tau/2} = \delta_x^2 u_{ij}^{n+1/2} + \delta_y^2 u_{ij}^n + \frac{\eta}{\tau/2}\Big(\delta_x^2 u_{ij}^{n+1/2} - \delta_x^2 u_{ij}^n + \delta_y^2 u_{ij}^n - \delta_y^2 u_{ij}^{n-1/2}\Big) + f_{ij}^n, \tag{5}$$

$$\frac{u_{ij}^{n+1} - u_{ij}^{n+1/2}}{\tau/2} = \delta_x^2 u_{ij}^{n+1/2} + \delta_y^2 u_{ij}^{n+1} + \frac{\eta}{\tau/2}\Big(\delta_x^2 u_{ij}^{n+1/2} - \delta_x^2 u_{ij}^n + \delta_y^2 u_{ij}^{n+1} - \delta_y^2 u_{ij}^{n+1/2}\Big) + f_{ij}^n, \tag{6}$$

where δ_x^2 and δ_y^2 are the standard approximations of the second derivatives, $i = \overline{1, N_1 - 1}$, $j = \overline{1, N_2 - 1}$, $h_1 = L_1/N_1$, $h_2 = L_2/N_2$. For each fixed value $j = 1, \ldots, N_2 - 1$, we solve equation (5) with nonlocal boundary conditions (3). For each fixed value $i = 1, \ldots, N_1 - 1$, we solve equation (6) with boundary conditions (4). In the case $\eta = 0$ this alternating direction method is Peaceman-Rachford method.

For investigation of stability we rewrite the three-layer scheme (5)-(6) in an equivalent form of a two-layer scheme $\mathbf{Y}^{n+1} = \mathbf{S}\mathbf{Y}^n$, where $\mathbf{Y}^{n+1} = (u^{n+1}, u^{n+1/2})^\top$, $\mathbf{S}$ is a nonsymmetric matrix of order $2(N_1 - 1)(N_2 - 1)$. The stability conditions are derived in a specially defined energy norm by investigating of the spectrum of $\mathbf{S}$ [1]-[2]. The stability of the difference scheme is investigated using a nonlinear eigenvalue problem.

REFERENCES

[1] M. Sapagovas, A. Štikonas, O. Štikonienė. Alternating direction method for the Poisson equation with variable weight cofficients in the integral condition. *Differ. Equ.*, **47** (8):1163–1174, 2011.

[2] M.Sapagovas. On the spectral properties of three-layer difference schemes for parabolic equations with nonlocal conditions. *Differ. Equ.*, **48** (7):1018-27, 2012..

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

MATHEMATICAL MODELING OF THE INFLUENCE OF INTERFERING SPECIES ON THE PERFORMANCE OF AN AMPEROMETRIC BIOSENSOR

LIANA STONKIENE

Institute of Informatics and Mathematics

Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: liana.stonkiene@mii.vu.lt

A mathematical model for an amperometric biosensors influenced by the presence of interfering species is based on nonlinear reaction–diffusion equations Various effects regarding amperometric bias caused by the presence of interfering species, have been analyzed. The model is solved with different initial and boundary conditions. The numerical simulation is carried out using the finite difference technique. The dynamics of the biosensor current are investigated. Using the proposed parameter values of the work [4] for the present model, the influence of the interfering species on the biosensor response was set.

REFERENCES

[1] R. Baronas, F. Ivanauskas, J. Kulys. Mathematical modeling of biosensors. *Vol. 9 of Springer Series on Chemical Sensors and Biosensors.* Springer, 2010.

[2] D. G. Buerk. Biosensors: theory and applications. *CRC Press.* Lancaster, 1995.

[3] R. Naujikas, A. Malinauskas, F. Ivanauskas. Modeling of electrocatalytic processes at conducting polymer modified electrodes. *Journal of Mathematical Chemistry*, **42** (4): 1069-1084, 2007.

[4] M. Puida, A. Malinauskas, F. Ivanauskas. Modelling of electrocatalysis at conducting polymer modified electrodes: nonlinear current-concentration profiles. *Journal of Mathematical Chemistry*, **49:** 1151-1162, 2011.

[5] A. A. Samarskii. The theory of difference schemes. *New York Basel:.* Marcel Dekker, 2001.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

PGD SPECTRAL SOLVER FOR SWIRL FLOWS [1]

ULDIS STRAUTINS

Institute of Mathematics and Computer Science, University of Latvia
Raiņa bulvāris 29, Rīga LV-1459, Latvia
Department of Physics and Mathematics, University of Latvia
Zeļļu iela 8, Rīga LV-1002, Latvia
E-mail: uldis.strautins@lu.lv

Swirl flows occur naturally in diverse transport, mixing, combustion processes in technology and nature. Consequently, optimizing the process wrt. the boundary conditions, distribution of external forces, geometry of domain and other parameters influencing the flow is of great interest. Often such flows are turbulent and require highly accurate methods, such as (pseudo)-spectral methods [1]. Predicting the flow for a range of values for several parameters in real time poses additional challenges which can be treated by PGD (proper generalized decomposition) techniques [2], [3].

For a domain $\Omega \subset \mathbb{R}^d$, consider the problem

$$u_t + u \cdot \nabla u - \mathrm{Re}^{-1} \triangle u = f - \nabla p,$$

with $\nabla \cdot u = 0$, initial condition $u|_{t=0} = u_0$ and Dirichlet BC $u|_{\partial\Omega} = g$.

We focus on an academic example where the flow domain is the unit square. Let a body force of the form $f = \sum \xi_i f_i^1(x) f_i^2(y)$ be applied on the fluid, with $\xi_i \in [a_i, b_i] : i = 1 \dots n$ real parameters. Solving the incompressible Navier-Stokes equations for representative sets of ξ_i is not realistic as the number of such sets grows exponentially with n.

A variant of PGD methods seeks the velocity and pressure fields at every time step t_n in a seperated representation, e.g., $u(x, y, \xi) = \sum_{i=1}^{N} X_i(x) Y_i(y) \Xi_i^1(\xi_1) \cdot \dots \cdot \Xi_i^n(\xi_n)$.

The set of basis functions X_i, Y_i, Ξ_i^j is enlarged iteratively wrt the index i, effectively exhausting the exact solution with a greedy algorithm. Integrating the equation wrt all but one of parameters, we obtain ODE boundary value problems for X_i and Y_i and algebraic (quadratic) equations for Ξ. Depending on the boundary conditions, Fourier or Chebyshev spectral methods can be applied to discretize the ODEs. In the present work we extend the approach in [3] to parameter dependent problems.

REFERENCES

[1] R. Peyret. *Spectral methods for incompressible viscous flow.* Springer Verlag, New York, 2002.

[2] A. Dumon, C. Allery and A. Ammar. Proper generalized decomposition (PGD) for the resulution of Navier-Stokes equations. *J Comp Phys*, **230** 1387–1407, 2011.

[3] A. Dumon, C. Allery and A. Ammar. Proper generalized decomposition for incompressible Navier-Stokes equations with a spectral discretization. *Appl Math Comp*, **219** 8145–8162, 2013.

[1] This work was partially supported by the grant 623/2014 of the Latvian Council of Science.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

NUMERICAL METHODS FOR SOLVING MULTI-TERM TIME-FRACTIONAL POISSON–NERNST–PLANCK EQUATIONS

I. TIMOSHCHENKO and N. ABRASHINA–ZHADAEVA

Belarusian State University

4 Independence av., Minsk, Belarus

E-mail: `igor_timoshchenko@yahoo.com, zhadaeva@bsu.by`

The time-fractional Poisson–Nernst–Planck equations (fPNPE) describe transport of charged particles through membrane possessing complex structure including internal buffers and traps that slow motion of ions. Numerical methods for solving fPNPE were proposed and studied in [1; 2].

However, different mechanisms of particle slowing can take place in the same membrane, and transport in such system can be adequately modeled by multi-term fractional time derivative $a_1\frac{\partial^{\beta_1}}{\partial t^{\beta_1}} + a_2\frac{\partial^{\beta_2}}{\partial t^{\beta_2}} + \dots$ [3]. In addition, if membrane considered with accompanying homogeneous diffusive layers, different order of time derivatives have to be used within membrane and these layers, and therefore coefficients a_i depend on coordinate.

In present work we study initial-boundary problem for equations

$$\sum_{i=1}^{n} a_i(x)\frac{\partial^{\beta_i} C_k}{\partial t^{\beta_i}} = D_k\frac{\partial^2 C_k}{\partial x^2} + D_k Z_k \frac{\partial}{\partial x}\left(C_k\frac{\partial \varphi}{\partial x}\right), \tag{1}$$

$$\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial \varphi}{\partial x}\right) = -\sum_{k=1}^{N_s} Z_k C_k - \rho_{fix}, \tag{2}$$

where C_k is concentration of $k-$th ion type, φ is electric field potential, $a_i \geqslant 0,\ 0 < \beta_i \leqslant 1,\ i = \overline{1,n}$,

$$\frac{\partial^\beta f(x,t)}{\partial t^\beta} = \frac{1}{\Gamma(1-\beta)}\int_0^t (t-\xi)^{-\beta}\frac{\partial}{\partial t} f(x,\xi)d\xi$$

is Caputo fractional derivative of order $\beta,\ 0 < \beta < 1$ [4].

We consider numerical methods for solution of equations (1)-(2) and its application.

REFERENCES

[1] I. Timoshchenko and N. Abrashina–Zhadaeva. Numerical algorithms for 1D fractional differential ionic transport problem in biological and synthetic membranes. In: *Abstracts of MMA2012*, June 6-9, 2012, Tallin, Estonia, 128.

[2] I. Timoshchenko and N. Abrashina–Zhadaeva. Implicit difference scheme for fractional Poisson–Nernst–Planck equations. In: *Abstracts of MMA2013*, March 27-30, 2013, Tartu, Estonia, 128.

[3] Y. Luchko. Initial-boundary-value problems for the generalized multi-term time-fractional diffusion equation. *Journal of Mathematical Analysis and Applications*, **374** (2):538-548, 2011.

[4] Anatoly A. Kilbas, Hari M. Srivastava and Juan J. Trujillo, North-Holland. *Theory and Applications of Fractional Differential Equations*. North-Holland Mathematics Studies, 204; ed. J. van Mill, Elsevier, Amsterdam, 2006.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

SIMULATION OF THREE-PHASE FLUID FLOW IN A POROUS MEDIUM USING EXPLICIT SCHEMES AND GPGPU COMPUTING

MARINA TRAPEZNIKOVA and NATALIA CHURBANOVA and ANASTASIYA LYUPA

Keldysh Institute of Applied Mathematics RAS

4 Miusskaya Square, 125047 Moscow, Russia

E-mail: `mtrapez@yandex.ru`
The work is supported by RFBR (grants No. 13-01-12073, 12-01-00556).

This work deals with further development of an original approach to porous media flow simulation using a kinetically-based model and explicit methods for its numerical implementation [1]. The governing model is derived by the analogy with the kinetically-consistent finite difference schemes and the related quasi-gas dynamic system of equations.

At present the model is generalized to the case of three-phase flow (water — non-aqueous phase liquid — gas), fluids are slightly compressible and immiscible, gravitational and capillary forces are taken into account. Due to the principle of minimal sizes in problems of continuum mechanics some minimal reference space and time scales exist and act as lower limits for the description details. Consequently the traditional continuity equation for a phase gets the regularizing term and the second order time derivative with small parameters. Thus the model involves the modified continuity equations, the generalized Darcy low and the state equations different for gas and liquid phases. The system is completed via relationships by Stone and by Parker for the relative phase permeability and capillary pressures, respectively. An algorithm of the explicit type is proposed: the hyperbolic continuity equation is approximated by the three-level explicit scheme with rather a mild stability condition, the convective term is approximated by central differences. The algorithm is easily adapted to hybrid supercomputers including graphics accelerators.

The parallel software library for modeling processes in the subsurface [2] is now supplemented with new computational modules and with the procedure of automatic data partitioning between processing units (CPUs or GPUs) to provide the optimal load distribution on the basis of a priory estimation of the run time.

Some test problems (like water infiltration into the impermeable box filled with sand saturated by water, oil and air) have been predicted. Computations are performed on hybrid system K100 with the 100 TFLOPS peak performance. The speedup of about 45 is achieved while a GPU is compared with a CPU core, the computational grid size is 6 million points. Weak scaling of the code has been investigated: the grid is enlarged correspondingly to the increased number of GPUs up to 36 million points calculated by six GPUs. Nearly linear growth of the speedup has been observed.

In the nearest future the created approach will be complicated by allowing for heat transmission in order to simulate thermal methods of oil recovery based on in-situ combustion.

REFERENCES

[1] D.N. Morozov, M.A. Trapeznikova, B.N. Chetverushkin and N.G. Churbanova. Application of Explicit Schemes for the Simulation of the Two Phase Filtration Process. *Mathematical Models and Computer Simulations*, **4** (1):62–67, 2012.

[2] D.N. Morozov, M.A. Trapeznikova, B.N. Chetverushkin and N.G. Churbanova. Simulation of Filtration Problems on Hybrid Computer Systems. *Mathematical Models and Computer Simulations*, **5** (3):208–212, 2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

EXPLICIT SCHEMES FOR PARABOLIC AND HYPERBOLIC EQUATIONS

PETR VABISHCHEVICH

Nuclear Safety Institute, Russian Academy of Sciences

52, B. Tulskaya, Moscow, Russia

North-Eastern Federal University

58, Belinskogo, Yakutsk, Russia

E-mail: vabishchevich@gmail.com

Standard explicit schemes for parabolic equations are not very convenient for computing practice due to the fact that they have strong restrictions on a time step. More promising explicit schemes are associated with explicit-implicit splitting of the problem operator (Saul'yev asymmetric schemes [1], explicit alternating direction (ADE) schemes [2], group explicit method [3]). These schemes belong to the class of unconditionally stable schemes, but they demonstrate bad approximation properties. These explicit schemes are treated as schemes of the alternating triangle method and can be considered as factorized schemes where the problem operator is splitted into the sum of two operators that are adjoint to each other.

Here we propose a multilevel modification of the alternating triangle method, which demonstrates better properties in terms of accuracy. We also consider explicit schemes of the alternating triangle method for the numerical solution of boundary value problems for hyperbolic equations of second order. The study is based on the general theory of stability (well-posedness) for operator-difference schemes.

Acknowledgements

This research was supported by RFBR (project 14-01-00785).

REFERENCES

[1] V.K. Saul'ev. *Integration of Equations of Parabolic Type by the Method of Nets.* Pergamon Press, New York, 1964.

[2] A.A. Samarskii. *The theory of difference schemes.* Marcel Dekker, New York, 2001.

[3] D.J. Evans. Alternating group explicit method for the diffusion equation. *Applied mathematical modelling*, **9** (3):201–206, 1985.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

ROBUST REGRESSION USING THE EMPIRICAL LIKELIHOOD METHOD

JANIS VALEINIS

Faculty of Physics and Mathematics, University of Latvia

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: `valeinis@lu.lv`

Empirical likelihood introduced by Owen [1] is a nonparametric method frequently used for statistical inference. Its advantage over classical methods is the Bartlet correctability and the asymmetry of the produced confidence intervals.

For different parametric, nonparametric and semiparametric regression problems many statistical methods have been developed using empirical likelihood (for overview see [2]). Recently Bondell and Stefanski [3] proposed the two-stage generalized empirical likelihood method. They propose to estimate the regression variance by some robust estimator and then to use that estimate within the framework of the generalized empirical likelihood method. According to their simulation study, the obtained regression estimators are both efficient and robust. We analyze their method for different simulated and real data problems. Moreover, we analyze the related ANOVA methods obtained from this procedure and compare with some classical ANOVA methods.

REFERENCES

[1] A. B. Owen. Empirical likelihood ratio confidence intervals for a single functional. *Biometrika*, **75** (2):237–249, 1988.

[2] X. S. Chen and I. Van Keilegom. A review on empirical likelihood methods for regression. *Test*, **18** (3):415–447, 2009.

[3] H. D. Bondell and L. A. Stefanski. Efficient robust regression via two-stage generalized empirical likelihood. *Journal of the American Statistical Association*, **108** (502):644–655, 2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

PARETO OPTIMIZATION FOR SECTORAL MODELS OF OPENCAST MINING

ANDREY M. VALUEV

Moscow State Mining University

MSMU, Leninsky prospect 6, Moscow 119991 Russia

E-mail: `valuev.online@gmail.com`

Search of Pareto points is one of possible ways of multiobjective optimization problems solution. The way it is applied to the problem in question is based on the following Theorem 1.

THEOREM 1. *If a vector $u^* \in R^n$ is a Pareto point for the problem*

$$F_i(u) \to \min, \quad i \in I_0, \qquad F_i(u) \le 0, \quad i \in I_1, \tag{1}$$

then for each $i_0 \in I_0$ the optimal value of target function in the problem

$$(F_{i_0u}(u^*), w) \to \min, \quad (F_{iu}(u^*), w) \le 0,\ i \in I_0 \cup I_{10}(u^*) \setminus \{i_0\}, \quad -1 \le w_j \le 1,\ i = 1, \cdots, n, \tag{2}$$

where $I_{10}(u) = \{i \in I_1 \mid F_i(u) = 0\}$ is zero.

For problems (1) based on the model [1] we have representation of $u = (u(1), \cdots, u(N))$, $u(k) \in R^{n_k}$, and of the problem itself

$$F_i(u(1), \cdots, u(N)) \to \min,\ i \in I_0, \qquad F_i(u(1), \cdots, u(N)) \le 0,\ i \in I_1(0), \tag{3}$$

$$F_i(u(k)) \le 0,\ i \in I_{11}(k), \quad F_i(u(k), u(k-1)) \le 0,\ i \in I_{12}(k), \quad k = 1, \cdots, N. \tag{4}$$

Problems (4) may be used to determine descent directions in Pareto optimization via feasible directions methods (FDM). Quasi-dynamic form of (3)–(4) enables (for a definite u) to decompose (2) into a set of $N + 1$ problems, dimension thereof being n_k, $k = 1, \cdots, N$, and $L \ll n$, by representation

$$\Delta u(k) = \alpha w(k) + C(k) \cdot \Delta u(k-1) + \alpha B(k) \cdot\ y(k) \tag{5}$$

where vector $y = (y(1), \cdots, y(N))$, $y \in R^L$, is expressed via linear functions depending on variations of the L so-called redundant (on u) restrictions caused by $w = (w(1), \cdots, w(N))$. Despite additional terms in (5) and extra calculations, the decomposition-based FDM diminishes the total amount of computations on an iteration for about $1/((L/n)^3 + 1/N^2 + (L/n)/N)$ times. For typical conditions ($N \sim 5$, $(L/n) \sim 0.1$) it may be evaluated by ~ 15. Treatment of $F_u(u(k))$, $F_u(u(k), u(k-1))$ sparsity serves as another source of computations efficiency.

REFERENCES

[1] A.M. Valuev. Methods of Open Pit Production Optimization Based on its Sectoral Models. In: *Abstracts of the 15th Int. Conf. MMA2010, May 26 - 29, 2010, Druskininkai, Lithuania*, Mathematical Modelling and Analysis, 2010, VSP, Vilnius, 109.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

THE RICCATI FAMILY OF DISTRIBUTIONS AND THE POLYLOGARITHM FUNCTION

MARK VAN LOKEREN

Vrije Universiteit Brussel

Pleinlaan 2, B-1050 Brussel, Belgium

E-mail: mvlokere@vub.ac.be

A random variable X is said to have a Riccati distribution if its cumulative distribution function $F(x)$ satisfies a Riccati equation. Assuming the coefficients of the Riccati equation to be constant, we determine closed-form expressions for the moments of X. We show that for any type of support of $F(x)$ (i.e. the real line, a semi-infinite interval or a finite interval) the moments can be expressed in terms of the polylogarithm function.

An interesting feature of our method is that it does not rely on the explicit expression of the cumulative distribution function $F(x)$ or the density function $f(x)$ (with $f(x)$ the derivative of $F(x)$). So there is no need to solve the Riccati equation in order to find the moments of the associated Riccati distribution. Moreover, our method allows the polylogarithm function to appear in a natural way through its definition as a repeated integral.

REFERENCES

[1] P. Jodrá. On a connection between the polylogarithm function and the Bass diffusion model.. *Proc. R. Soc. A*, **(464):** 3081–3088, 2008.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

SOLUTIONS OF THE SYSTEM OF EQUATIONS TO THE COMPLEX-VALUED PENDULUM

INARA YERMACHENKO

Department of Mathematics, Daugavpils University

Parādes iela 1, Daugavpils LV-5400, Latvia

E-mail: inara.jermacenko@du.lv

The equation of simple pendulum given by $z'' = -a \sin z, \quad a > 0$ in a case of complex-valued function $z(t) = x(t) + i\, y(t)$ gives rise to a coupled nonlinear system

$$\begin{cases} x'' = & -a \sin x \cosh y, \\ y'' = & -a \cos x \sinh y. \end{cases} \tag{1}$$

Solutions of the system (1) and their properties are investigated considering existing constants of a motion and different boundary conditions.

REFERENCES

[1] S. S. Abdullaev . *Construction of Mappings for Hamiltonian Systems and Their Applications* . Lect. Notes Phys. 691 (Springer, 2006) , ISBN-10 3-540-30915-2 .

[2] A. Beléndez, C. Pascual, D. I. Méndez, T. Beléndez and C. Neipp. Exact solution for the nonlinear pendulum. *Revista Brasileira de Ensino de Física, 2007* , **vol. 29, n. 4,** p.645-648.

[3] S. Bhalekar . Infinite-scroll attractor generated by complex pendulum model. *Inter. J. of Analysis, Hindawi* , **vol. 2013,** article ID 368150, http://dx.doi.org/10.1155/2013/368150.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

MECHANICS-MATHEMATICAL MODEL FOR THE DETERMINATION OF PHYSICAL AND MECHANICAL BIOCELLS PROPERTIES

MICHAEL ZHURAVKOV and NATALIE ROMANOVA

Belarusian State University

Nezavisimosty 4, BY-220030, Minsk, Belarus

E-mail: Zhuravkov@bsu.by, Romanovans@bsu.by

Determination of physical and mechanical biological tissue (cells) properties is one of the most important scientific processes whose results affect the drawing up description of a real situation of human health state. Mechanical properties of biotissue are changed in the vital activity process [1]. Thus various types of biological objects have different mechanical properties which dependent on cell cycle-phase, cellular-level activity, environmental and a number of physical and chemical factors. Construction of the correct mechanics-mathematical models for a biocells behaviour description is significant for medical research and disease diagnosis [2].

Cellular mechanical properties can be described using elasticity, viscosity and creep compliance. It is important to observe that behaviour of many biological composites is similar because biological cell is considered as viscoelastic material. This fact is especially significant for carrying out literary analysis of cell mechanical properties data (that have a sufficiently greater variance). In general case creep and relaxation processes in real heterogeneous media are nonlinear in both space and time. Thus the use of fractional order derivatives in state equation of viscoelastic media allows to take into account as heterogeneous structures of both viscous and elastic elements as non-uniform mechanical processes in time. Nowadays fractional calculus is successfully used applied in this line [3] .

Modification of Hertz contact problem is offered to use for the determination of physical and mechanical biotissue properties. Mechanics-mathematical model was designed on basis of the fractional calculus. In particular, in present research the relaxation function was used in the following forms:

– Rabotnov fractional exponential kernel [4] $\mu_\alpha(-1,t) = t^\alpha \sum\limits_{n=0}^{\inf} \frac{(-1)^n t^n (1+\alpha)}{\Gamma[(n+1)(1+\alpha)]}$, which is resolvent of Abel's creep kernel $I_\alpha(t) = \frac{t^\alpha}{\Gamma(1+\alpha)} (-1 < \alpha < 0)$

– exponential relaxation kernel $R(t) = Be^{-qt}$, which is resolvent of creep kernel $\Gamma(t) = Ae^{-pt} (A > 0, p > 0)$

REFERENCES

[1] Y.C. Fung. *Biomechanics: Mechanical Properties of Living Tissues.* New York, Springer-Verlag, 1993.

[2] M.Zhuravkov and N. Romanova. Fractional models for determination of physical and mechanical properties biological tissue. In: *9th International ISAAC Congress,August 5-9, Krakow,Poland*, 2013, .

[3] M.Zhuravkov and N.Romanova. Fractional calculus application prospects in mechanics,Minsk: BSU. In: *http://elib.bsu.by/handle/123456789/37576*, , .

[4] Y.Rabotnov. *Creep of structural elements.* Moskow, Nauka, 1966. (in Russian)

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

NON-CONVEX MULTI-OBJECTIVE OPTIMIZATION

JULIUS ŽILINSKAS

Vilnius University Institute of Mathematics and Informatics

Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: julius.zilinskas@mii.vu.lt

Real-world optimization problems usually involve more than one criteria, what leads to solution of multi-objective optimization problems. Such a kind of optimization problems are important in various fields of industry and research.

A multi-objective optimization problem with d objectives $f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_d(\mathbf{x})$ is to minimize the objective vector $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_d(\mathbf{x}))$:

$$\min_{\mathbf{x} \in X} \mathbf{f}(\mathbf{x}),$$

where $\mathbf{x}$ is the decision vector and X is the search space. In most cases it is impossible to minimize all objectives at the same time, so there is no single optimal solution to a given multi-objective optimization problem. Therefore non-dominated "Pareto-optimal" solutions are searched.

Multi-objective optimization is a research area rich with various approaches [1; 2; 3]. Many methods convert the multi-objective optimization problem into a set of single-objective problems. The most known methods are the linear scalarization where objectives are aggregated with positive weights and the ε-constraint method where one objective is minimized while the others are converted to constraints. Apart from general disadvantages of such approaches, in non-convex multi-objective optimization even the scalarized single-objective optimization problem is not easily solved – global optimization must be used.

Branch and bound method can be used to solve global optimization problems. Although it could be possible to solve non-convex multi-objective optimization problem solving a set of global optimization problems by branch and bound algorithm, a more advantageous approach is to develop multi-objective branch and bound algorithms which solve multi-objective optimization problem in one run. In this lecture we demonstrate this approach and show its advantages. In non-convex continuous multi-objective optimization such a multi-objective branch and bound algorithm can find close approximation of the Pareto front with predefined accuracy and in discrete multi-objective optimization the exact Pareto-optimal set can be found.

Acknowledgments.

This research was funded by a grant (No. MIP-063/2012) from the Research Council of Lithuania.

REFERENCES

[1] K. Deb. *Multi-objective Optimization Using Evolutionary Algorithms.* John Wiley & Sons, Chichester, 2009.

[2] K. Miettinen. *Nonlinear Multi-objective Optimization.* Kluwer Academic Publishers, Boston, 1999.

[3] C. Zopounidis and P. M. Pardalos (Eds.). *Handbook of Multicriteria Analysis.* Springer-Verlag, Berlin Heidelberg, 2010.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

SPLITTING HIGHER ORDER SCHEMES WITH DISCRETE TRANSPARENT BOUNDARY CONDITIONS FOR THE SCHRÖDINGER EQUATION IN AN INFINITE PARALLELEPIPED

ALEXANDER ZLOTNIK and ALLA ROMANOVA

Department of Higher Mathematics at Faculty of Economics, National Research University Higher School of Economics

Myasnitskaya 20, 101000 Moscow, Russia

E-mail: azlotnik2008@gmail.com; avromm1@gmail.com

Solving the n-dimensional time-dependent Schrödinger equation in unbounded domains is important in many fields. We consider an initial-boundary value problem for this equation in an infinite (or semi-infinite) parallelepiped ($n \geqslant 2$) and, in particular, in a strip for $n = 2$. Starting from the Numerov-Crank-Nicolson finite-difference scheme for $n = 2$ [1], we firstly construct another higher order scheme with splitting space averages having much better spectral properties for $n \geqslant 3$. Secondly, we apply the three stage Strang-type splitting in time with respect to the potential both to the Numerov-Crank-Nicolson scheme for $n = 2$ and the latter one for $n \geqslant 2$. The both constructed schemes have the approximation order $O(\tau^2 + |h|^4)$. To begin with, we consider them on the uniform infinite space mesh in the infinite parallelepiped and prove the unconditional uniform in time L^2-stability together with the mass conservation law.

Thirdly, we construct discrete transparent boundary conditions (DTBCs) at artificial boundaries. The DTBCs allow to restrict rigorously the solution of the scheme on the infinite space mesh to a finite one. Among other artificial boundary conditions, their advantages are the complete absence of spurious reflections, reliable computational stability, clear mathematical background and corresponding rigorous stability theory. The explicit form of the DTBCs is non-local and involves the discrete convolution in time together with the discrete Fourier transforms in space directions perpendicular to the leading axis of the parallelepiped. We derive the uniqueness of solution and the uniform in time L^2-stability for both splitting schemes with the DTBCs.

Due to splitting, an effective direct algorithm is developed to implement the schemes for general potential using FFT in the perpendicular directions. The corresponding numerical results on the tunnel effect for both smooth and (discontinuous) rectangular barriers are presented for $n = 2$. They are accompanied by the practical error analysis using the numerical solutions on refining meshes in space and time directions. Even in the case of the rectangular barriers, the numerical results improve previous ones by taking coarser space mesh. The results are presented in detail in [2; 3].

REFERENCES

[1] M. Schulte and A. Arnold. Discrete transparent boundary conditions for the Schrödinger equation, a compact higher order scheme. *Kinetic Relat. Models*, **1** (1): 101–125, 2008.

[2] A. Zlotnik and A. Romanova. A Numerov-Crank-Nicolson-Strang scheme with discrete transparent boundary conditions for the Schrodinger equation on a semi-infinite strip. *http://arxiv.org/abs/1307.5398*, 2013.

[3] B. Ducomet, A. Zlotnik and A. Romanova. A splitting higher order scheme with discrete transparent boundary conditions for the Schrodinger equation in a semi-infinite parallelepiped. *http://arxiv.org/abs/1309.7280*, 2013.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

PROFIT-MAXIMIZING PRODUCTION MODEL WITH STOCK-DEPENDENT DEMAND AND TIME-DEPENDENT HOLDING COST

HESHAM K. ALFARES, AHMAD A. AL-SHAER

Systems Engineering Department, King Fahd University of Petroleum and Minerals

Dhahran 31261, Saudi Arabia

E-mail: alfares@kfupm.edu.sa

The objective of this paper is to develop mathematical models and optimal solution procedures that maximize the total profit for a production-inventory system with stock-level-dependent demand rate, finite production rate, and two types of variable holding cost. The new model eliminates several limitations of traditional inventory models, namely the assumptions of constant demand, constant holding cost, instantaneous order arrival, minimum-cost objective, and zero starting inventory. The main features of the proposed model are: (1) production rate is a finite and known constant, (2) demand rate is a function of the current inventory level, (3) holding cost is a function of storage time, (4) the initial inventory is a decision variable, and (5) the objective is to maximize total profit. Two models are presented that consider two different structures for the unit holding cost increase with longer storage duration: retroactive increase, and incremental increase.

Unlike traditional inventory models with constant demand, the proposed system assumes that the total demand - and hence total sales revenue - is stock-level dependent that is affected by the inventory decisions made. Therefore, the objective of the model is to maximize profits rather than minimize costs. Pursuing a profit-maximization objective may lead to higher inventory levels to increase sales and potentially obtain a greater profit. Consequently, it is more realistic to allow the system to have a positive inventory at the start and end of the order cycle. For the proposed production-inventory system, mathematical programming models are formulated, and optimum solution algorithms are developed.

Abstracts of MMA2014, May 26 - 29, 2014, Druskininkai, Lithuania

AN ADI EXTENSION FOR NODAL NUMERICAL SOLUTION OF 2D HELMHOLTZ EQUATION

PATRIKS MOREVS[1], JANIS RIMSHANS[1] and MAKSIMS ZIGUNOVS[1]

[1] *Institute of Mathematical Sciences and Information Technologies, Liepaja University*

Liela Street 14, Liepaja LV-3401, Latvia

E-mail: `patriks.morevs@liepu.lv`, `janis.rimsans@liepu.lv`, `maksims.zigunovs@liepu.lv`

The numerical solution algorithm for the propsed nodal difference scheme [1] is elaborated by application of an ADI based iteration method, which allows use of HPC for the 2D Helmholtz equation, $(x, y) \in \Omega$, with the corresponding boundary conditions in $\partial\Omega$:

$$G_x \partial_x \left(D_x \partial_x \Phi \left(x, y \right) \right) + G_y \partial_y \left(D_y \partial_y \Phi \left(x, y \right) \right) + \sigma \Phi \left(x, y \right) = 0. \tag{1}$$

The Neumann convergence condition for solutions of the equation (1) shows conditional convergence. The example of an eigenvalue and eigenfunction problem from [2] was used to test the parallel numerical computation algorithm. It is shown that the elaborated numerical approach has the 2nd order of precision. As it is shown in Table 1, the ADI computations have good results in respect to the computation time and absolute error for these sparse matrices.

Table 1.
The calculated absolute error and CPU time (t) in L2 norm

Grid size	ADI method, t (s)	Absolute error	LSCOV (including LU decomposition), t (s)
128x128	2	$2.64 * 10^{-5}$	10
256x256	15	$5.65 * 10^{-6}$	57
512x512	99	$1.34 * 10^{-6}$	254
1024x1024	667	$3.28 * 10^{-7}$	—
2048x2048	7350	$7.90 * 10^{-8}$	—

The computation time of ADI type method was at least 2 times shorter in comparison to the computation time needed by solving the system of linear equations applying a well-known iteration method *LSCOV*. In the case of using *LSCOV* for grids of size larger than 512x512 the results could not be computed. Therefore the advantage of the ADI type method is the possibility to obtain results in the case of very large grids with the desired precision. However, the application of the method requires some additional research to be applied for larger wave numbers.

REFERENCES

[1] P. Morevs, J. Rimshans and Sh.E. Guseynov. Nodal Numerical 2D Helmholtz Equation: Truncation Analysis. *Advanced Material Research*, **222.** 345-348, 2011.

[2] R. Courant, D. Hilbert. Methods of Mathematical Physics. *Physics of Plasmas*, **Vol1.** London, 1966.

Index