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ON A LOGISTIC FUZZY DIFFERENCE EQUATION

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Difference equations appear as a natural way of evolution phenomena because most measurements of time evolving variables are discrete and these equations are important in mathematical models. More importantly, difference equations also appear in the study of discretization methods for differential equations.

The logistic difference equation was popularized in a seminal 1976 paper by the biologist Robert May [2], as a discrete-time demographic model analogous to the logistic equation first created by Pierre Franois Verhulst [1]. The discrete version of the logistic model is written as

$$x_{n+1} = ax_n(1 - x_n), \quad n = 0, 1, ... \tag{1}$$

We mention that if $a < 1$, the model describes extinction of population.

In this talk we consider the fuzzy difference equations

$$x_{n+1} = \beta x_n(1 - x_n), \tag{2}$$

and

$$x_{n+1} = \beta x_n \ominus \beta {x_n}^2, \quad n = 0, 1, ..., \tag{3}$$

where x_n is a sequence of positive fuzzy numbers, β and initial value x_0 are positive fuzzy numbers and $\ominus$ denotes the Hukuhara difference (H-difference) of two fuzzy numbers.

Proposition 1. *Consider Eq. (2) where x_n is a sequence of positive fuzzy numbers and $x_0, \beta \in \mathbb{R}^+_{\mathcal{F}}$. If $\overline{x_{0,\alpha}}, \overline{\beta_\alpha} < 1, \quad \forall \alpha \in (0, 1]$, then for every positive fuzzy number x_0, there exists a unique positive solution x_n of (2) with initial condition x_0.*

Proposition 2. *Consider Eq. (3) where x_n is a sequence of positive fuzzy numbers and $\beta, x_0 \in \mathbb{R}^+_{\mathcal{F}}$ such that $\overline{\beta_\alpha} < 1, \quad \overline{x}_{0,\alpha} \leq \frac{1}{2}, \quad \forall \alpha \in (0, 1]$. Then for every positive fuzzy number x_0, there exists a unique positive solution x_n of (3).*

REFERENCES

[1] N. Bacar. *A Short History of Mathematical Population Dynamics*. Springer, Berlin, 2011.

[2] R. May . Simple mathematical models with very complicated dynamics. *Nature* , **261** (3):459–467, 1976.

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GENERALIZED SMOOTHING SPLINES AND CONTROL THEORY

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The original idea of control theoretic splines (see, e.g., [2], [3]) is to use a controlled differential equation to push a curve nearby a set of desired points

$$\{(t_i, z_i) : i = 1, 2, \ldots, n\}, \quad \text{where} \quad a \le t_1 < t_2 < \ldots < t_n \le b.$$

In this report we deal with the linear system

$$x' = Mx + \beta u, \quad y = \gamma^\top x, \tag{1}$$

where x is a vector-valued absolutely continuous function defined on $[a, b]$, M is a given matrix and β, γ are given vectors of compatible dimensions. We consider system (1) as the curve $z = y(t)$ generator. The goal is to find a control law $u \in L_2[a, b]$ that minimizes the quadratic cost function

$$\int_a^b u^2(t)dt + \rho \sum_{i=1}^{n} (y(t_i) - z_i)^2 \longrightarrow \min \tag{2}$$

with a positive weight ρ. In some applications problem (1)-(2) is supplemented with additional constraints on its solutions.

Let us note that in the scalar case (with real-valued x) a solution of minimization problem (2) under $x'' = u,\ y = x$, can be obtained using the corresponding cubic smoothing spline s by $u = s''$.

The aim of this report is to show how the technique of generalized smoothing splines [1] can be adapted for construction of control theoretic splines. This study is closely related to our previous work [1] and shows possible applications of smoothing splines with additional restrictions in the control theory.

REFERENCES

[1] S. Asmuss and N. Budkina. On Some Generalization of Smoothing Problems. *Mathematical Modelling and Analysis*, **15** (3):311–328, 2015.

[2] M. Egerstedt and C. Martin. *Control Theoretic Splines*. Princeton University Press, 2010.

[3] J. Shen and X. Wang. A Constrained Optimal Control Approach to Smoothing Splines. In: *Proceedings of IEEE Conference on Decision and Control*, Orlando, 2011, pp. 1729–1734 .

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MODELLING OF SCANNING ELECTROCHEMICAL MICROSCOPY USING DIFFUSION AND REACTION EQUATIONS

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Scanning electrochemical microscopy (SECM) is an advanced electrochemical method, which is based on scanning of the enzyme-modified surface by an ultramicroelectrode. Due to high complexity of such systems and small scale, various mathematical models are developed of processes that are influencing the signal of SECM [1].

We analysed redox-competition mode where oxygen is consumed both by the electrode and by reactions on the surface [2]. We developed the 2-dimensional time-dependable model in cylindrical coordinates representing diffusion and reactions between eight most important reagents. The system of 8 diffusion and reaction equations was presented. The boundary conditions for diffusion equations on the enzyme-modified surface were formulated in accordance with chemical reactions.

The described system of equations was solved numerically using alternating-direction finite difference and iterative methods. Non-uniform grid was chosen because of very small size of the electrode compared to entire domain and to decrease the computational error near the corners of the electrode. Numerical simulation results were compared with the experimental data. Model parameters such as reaction rate constants were calculated by fitting numerical data to experimental. Very good match between experimental and numerical results was achieved by setting oxygen diffusion coefficient according to the density of the surrounding medium.

REFERENCES

[1] R. Cornut, C. Lefrou. A unified new analytical approximation for negative feedback currents with a microdisk SECM tip. *Journal of Electroanalytical Chemistry*, **608 1:** 5966, 2007.

[2] F. Ivanauskas, I. Morkvenaite-Vilkonciene, R. Astrauskas, A. Ramanavicius. Modelling of Scanning Electrochemical Microscopy at Redox Competition Mode Using Diffusion and Reaction Equations. *Electrochimica Acta*, **222:** 347-354, 2016.

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PARALLEL SOLUTION METHODS FOR EVOLUTION EQUATIONS

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Traditional solution methods for time-dependent partial differential equations are based on time-stepping methods, explicit or implicit. For reasons of numerical stability explicit methods require use of very small time-steps. For implicit methods one must also choose a somewhat small time-step to achieve a sufficiently small time discretization error. Furthermore, each step is costly to execute as it requires the solution of a normally elliptic, even though scaled,equation on each time interval. Clearly, the methods are sequential. Therefore they are costly and time-consuming. However, there exit alternatives to time-stepping methods.

In many practically important problems, such as in optimal control problems, the control function is periodic, for instance an alternating current in electromagnetic problems. Then the problem becomes time-harmonic and the solution can be approximated with a truncated Fourier series expansion. Due to the orthogonality of the trigonometric functions, for linear problems the computation of the Fourier coefficients separate and one can compute the solution for each period fully in parallel. Hence there is a perfect scalability of the solution process.

For nonlinear problems one can use a two-grid method, i.e. solve the nonlinear equation on a coarse grid, interpolate the solution on the fine grid and compute a corrected solution of the linearized Newton type equation just once.

For each frequency problem a system on two-by-two or four-by-four block matrix arises. In a series of publications it has been demonstrated that for such problems a very efficient preconditioner, the preconditioned square matrix block, PRESB method exists and leads to very tight eigenvalue bounds and hence few iterations. This holds uniformly with respect to mesh size, control cost and other regularization parameters as well as to the problem parameters. The method has been shown to outperform other methods for the problems considered.

For more general problems than time-harmonic problems one can replace the trigonometric functions with orthogonal polynomials, such as based on Legendre polynomials. This leads to a similar but somewhat more complicated system to be solved than for the time-harmonic method, but it can also to some extent be performed in parallel.

Several numerical test examples will be presented.

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MIXED-STABLE MODELLING OF LARGE SETS OF HIGH-FREQUENCY FINANCIAL DATA

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Modern electronic trading provides economists and market analysts with a large quantity of trading data. High-frequency and intra-daily data contain all transactions of the financial market and can reveal events and laws impossible to uncover with monthly, weekly or even daily data [3]. However, all that comes with its price. The processing time increases with the amount of the financial data. Meanwhile in business decision-making the time is crucial. Processing huge amounts of high-frequency data is virtually impossible without the application of modern parallel technologies.

The increased availability of high-frequency data has caused a great interest in the research of this subject. The main applications are in financial engineering, ranging from risk management to options hedging, execution of transactions, portfolio optimization, and forecasting. Methodologies based on the high-frequency data analysis can also be found in neural science and the real-time network traffic management.

A common perception is that the most accurate method of processing - maximum likelihood estimation - is very time-consuming. Therefore it is often rejected. However, as we show, the application of parallel technologies makes this method both precise and practical [1]. In this research, we apply our parallel computing approach [2] to the mixed-stable modelling of high-frequency data. In practice we often observe a large number of zero returns in the high-frequency return data due to the fact that the underlying asset price does not change at given short-time intervals. The mixed-stable model is designed to cope with these specific features, often observed in the high-frequency return data.

The purpose of this research is to investigate the application of the mixed-stable model for the analysis of large sets of high-frequency financial data (specifically yearly German DAX stock index log-returns series). The insufficiency of the classical Gaussian as well as standard α-stable models is demonstrated. Mixed-stable parameters are estimated numerically with the maximum likelihood method. Efficient parallel algorithms are employed for the processing of long-term data series. We have studied the influence of the accuracy of probability density function calculation and maximum likelihood optimization on the results of the modelling and processing time. Mixed-stable models for all 29 DAX companies are constructed. The adequacy of models is verified with empirical characteristic functions goodness-of-fit test. Obtained mixed-stable parameter estimates can be used for the construction of the optimal asset portfolio.

REFERENCES

[1] I. Belovas, V. Starikovičius. Parallel computing for mixed-stable modelling of large data sets. *Information Technology and Control*, **44** (2):148–154, 2015.

[2] I. Belovas, V. Starikovičius. Parallelization of α-stable modelling algorithms. *Mathematical modelling and analysis*, **12** (4):409–418, 2007.

[3] R. Dacorogna et al. *An Introduction of High-Frequency Finance*. Academic Press, San Diego, 2001.

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GREEN'S FUNCTION METHOD FOR AN ELEMENT WITH CYLINDRICAL RODS

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As modern computer systems are required to compute more, increased cooling is needed to mitigate the thermal issues and to remove the excess heat produced by computer components. One way to improve heat transfer is to increase surface area by using extensions (called fins) on the surface of the components. In the literature (e.g. [1], [2]), many studies exist on heat transfer in fins and fin assemblies. In the article [3], the authors examine a system with cylindrical shaped fins and propose a mathematical model for one-dimensional steady state heat conduction. However, the exact solution is not provided.

Over the past few years we have been generalizing Green's function method for domains of non-canonical form (see [4] – [6]) for solving different types of partial differential equations. Here we consider three-dimensional heat transfer problem in a rectangular plate with cylindrical fins. By means of the Green's function method we construct a solution of the problem. An analytical approximate solution is derived using conservative averaging method.

REFERENCES

[1] D. Q. Kern, A. D. Kraus. *Extended Surface Heat Transfer*. McGraw-Hill, New York, 1972.

[2] M. Manzoor. *Heat Flow Through Extended Surface Heat Exchangers*. Springer-Verlag: Berlin and New York, 1984.

[3] L. A. Beltrán-Prieto, J. C. Beltrán-Prieto, Z. Komínková-Oplatková. Mathematical model of an integrated circuit cooling through cylindrical rods. *The 2016 International Conference Applied Mathematics, Computational Science and Systems Engineering*, **9,** 2017.

[4] M. Lencmane, A. Buikis. Article in proceedings. In: *Proceedings of the 9th IASME/WSEAS International Conference on HEAT TRANSFER, THERMAL ENGINEERING and ENVIRONMENT (HTE11), Florence, Italy, 2011*, Analytical Solution of a Two-Dimensional Double-Fin Assembly, WSEAS Press, 2011, 396 – 401.

[5] T. Bobinska, M. Buike, A. Buikis. Article in proceedings. In: *Proceedings of the 9th IASME/WSEAS International Conference on HEAT TRANSFER, THERMAL ENGINEERING and ENVIRONMENT (HTE11), Florence, Italy, 2011*, Comparing Solutions of Hyperbolic and Parabolic Heat Conduction Equations for L-shape Samples, WSEAS Press, 2011, 384 – 389.

[6] S. Blomkalna, M. Buike, A. Buikis. Article in proceedings. In: *Proceedings of the 9th IASME/WSEAS International Conference on HEAT TRANSFER, THERMAL ENGINEERING and ENVIRONMENT (HTE11), Florence, Italy, 2011*, Several Intensive Steel Quenching Models for Rectangular and Spherical Samples, WSEAS Press, 2011, 390 – 395.

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ON SOME FEATURES OF SYNTHETIC MEASURES

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The main goal of a taxonomic analysis is to group and to order objects that are elements of a multidimensional feature space, and different methods of such analysis were developed (see, for example, [1; 2]). It starts usually from a reduction of a large quantity of gathered data about the objects to a few basic synthetically described categories or synthetic measures of their features that are a subject of further analysis. From the other side, an inverse problem may be essential when the ordering or grouping of objects is known and we are looking for the objects features and their measures that have led to such ordering or grouping.

According to Hadamard, a problem in natural sciences has been formulated correctly if:

- there exists its solution satisfying some given conditions;
- the solution is unique;
- the solution depends continuously on the defined conditions (it is stable).

On the other hand, in economic and social sciences correctness of a problem formulation is interpreted in different ways. According to D.H. Jackson [1], for instance, a problem has been formulated correctly if:

- an application of an algorithm results in a single solution;
- the resulting classification (ordering) is stable. It means that a small variation of input data results in a small change of classification;
- the applied algorithm is invariant under permutations of objects classified by their names.
- the applied algorithm does not depend on the scale transformation.

These conditions are definitively not sufficient because they omit entirely the problem of unsolved issues of adequateness of configurations of individual classes in relationship to specific connections between objects or to specific needs. For over ten years the authors of this paper have been working on constructing new measures of classification and grouping of objects, as well as new measures of concentration - analogous to Gini Index and Herfindahl-Hirschman Index (see [3]). Consequently, the authors have designed various constructions of measures: based on reference objects, utilizing radar charts of vectors. They have also provided new ways of calculating measures of similarity between objects, which are analogues of known coefficients of Jaccard, Dice, Tanimoto and Tversky. This work is a part of that cycle and contains basic properties of previously studied measures.

REFERENCES

[1] D.H. Jackson. *Comparison of classification.* Academic Press, 1969.

[2] L. Breiman. Bias-variance, regularization, instability and stabilization. In: *Neural Networks and Machine Learning*, C.M. Bishop (Ed.), Springer-Verlag, Berlin, 1998, 27 – 56.

[3] Z. Binderman, B. Borkowski, A. Prokopenya, and W. Szczesny. Mechanisms of construction of the radar coefficients of concentration. In: *Computer Algebra Systems in Teaching and Research*, vol. IV, No. 1, Wydawnistwo Collegium Mazovia, Siedlce, 2013, 107 – 116.

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STABILITY OF IMPLICIT-EXPLICIT GENERAL LINEAR METHODS

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Semi-discretization in space of time-dependent partial differential equations often leads to large systems of ordinary differential equations for which there are natural splittings into non-stiff (or mildly stiff) and stiff parts. Such systems can be written in the form

$$\begin{cases} y'(t) &= f(y(t)) + g(y(t)), \quad t \in [t_0, T], \\ y(t_0) &= y_0 \in \mathbb{R}^m, \end{cases} \tag{1}$$

where $f\colon \mathbb{R}^m \to \mathbb{R}^m$ represents non-stiff part and $g\colon \mathbb{R}^m \to \mathbb{R}^m$ stiff part of the system.

System (1) can be solved efficiently by implicit-explicit (IMEX) schemes (see [1] and references therein). In present research we consider the class of general linear methods (GLMs, [2]). We construct IMEX GLMs of order $p = 1, 2, \ldots, 5$ with large regions of absolute stability of the explicit part of the method, assuming that the implicit part of the method is A-stable. Next, we analyze convergence and error of methods. We also present results of numerical experiments.

This is a joint work with G. Izzo and Z. Jackiewicz.

REFERENCES

[1] M. Braś , G. Izzo and Z. Jackiewicz. Accurate Implicit-Explicit General Linear Methods with Inherent Runge-Kutta Stability. *J. Sci. Comput.*, **70** (3):1105–1143, 2017.

[2] Z. Jackiewicz. *General Linear Methods for Ordinary Differential Equations*. John Wiley, Hoboken, New Jersey, 2009.

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THE PARALLEL OPTIMIZATION ALGORITHM FOR PROBLEMS INVOLVING SIMULTANEOUS SOLUTION OF DIFFERENT DIFFERENTIAL EQUATIONS

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In this work we consider a general class of global optimization problems for which the computation of objective function requires solution of M different subproblems, with a priori estimated subproblems sizes $Z_i, i = 1, \ldots, M$. Such optimization problems are often very computationally intensive. Thus, the parallel computations are necessary. In general case we highlight three levels of parallelization:

1. Parallel local minimum calculation for different starting points. However, if, e.g., evolutionary algorithms are used, then it limits this level, since we need to serialize solution at different points or groups of points (i.e. next point is determined by some previous ones). Also, the number of simultaneously calculated points must be much larger than the number of processes to avoid load balancing issues, since sizes of computations can vary depending on a starting point.
2. Calculation of solutions for different independent subproblems in parallel. Note, that sizes of different mathematical subproblems can be different.
3. Parallel solution of each subproblems and parallel calculation of objective function. Each subproblem is solved by a group of parallel processes. The size of each group is selected according to the estimate of subproblem's size Z_i.

More specifically, in our research we concentrate on a special case, when each subproblem involves solving differential equation that can be numerically described as a system of linear equations with tridiagonal matrix. As an example of such optimization problem we take a problem that is formulated in [2]. There the linear one-dimensional Schrödinger equation is solved using the approximation of boundary conditions by rational functions [1]. We use the combination of errors for different test cases as an objective function. As a local optimizer simplex downhill method is used. To solve the discrete approximation of differential equation in parallel we apply Wang's algorithm [3]. We focus on the last two levels of parallelization, since the first one is usually trivial in terms of parallelization. Computational experiments were performed on "HPC Sauletekis".

REFERENCES

[1] X. Antoine and A. Arnold and C. Besse and M. Ehrhardt and A. Schädle. A review of transparent and artificial boundary conditions technique for linear and nonlinear Schrödinger equations. *Commun. Comput. Phys.*, **4** (4):729–793, 2004.

[2] A. Bugajev, R. Čiegis, R. Kriauzienė, T. Leonavičienė and J. Žilinskas (in press). On the accuracy of some absorbing boundary conditions for the Schrödinger equation. *Mathematical modelling and analysis*, **22** (3), 2017.

[3] H.H. Wang. A parallel method for tridiagonal equations. *ACM Transactions on Mathematical Software (TOMS)*, **7** (2):170–183, 1981.

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GREEN'S FUNCTION METHOD FOR CONNECTED REGULAR DOMAINS

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Greens functions have long been used to create integral representations for boundary value problems. Although the method is usually associated with canonical domains [1], we propose to use this method for non-canonical adjacent domains. We generalize the Greens function method by conjugation conditions along the surface connecting two neighbour domains. This generalization is being used for elliptic, parabolic and hyperbolic types of partial differential equations [2] - [6].

REFERENCES

[1] G. F. Roach. *Greens Functions.* Cambridge University Press, 1982.

[2] M. Buike, A. Buikis. Article in proceedings. In: *Applied and Computational Mathematics. Proceedings of the 13th WSEAS International Conference on Applied Mathematics (MATH08), Puerto De La Cruz, Tenerife, Canary Islands, Spain, December 15-17, 2008*, Hyperbolic Heat Equation as Mathematical Model for Steel Quenching of L-Shape Samples, Part 1 (Direct Problem), WSEAS Press, 2008, 198 – 203.

[3] T. Bobinska, M. Buike, A. Buikis. Article in proceedings. In: *Proceedings of the 5th IASME/WSEAS International Conference on CONTINUUM MECHANICS (CM10), Cambridge, UK, 2010*, Hyperbolic Heat Equation as Mathematical Model for Steel Quenching of L-Shape Samples, Part 2 (Inverse Problem), WSEAS Press, 2010, 21 – 26.

[4] M. Lencmane, A. Buikis. Article in proceedings. In: *Proceedings of the 9th IASME/WSEAS International Conference on HEAT TRANSFER, THERMAL ENGINEERING and ENVIRONMENT (HTE11), Florence, Italy, 2011*, Analytical Solution of a Two-Dimensional Double-Fin Assembly, WSEAS Press, 2011, 396 – 401.

[5] T. Bobinska, M. Buike, A. Buikis. Article in proceedings. In: *Proceedings of the 9th IASME/WSEAS International Conference on HEAT TRANSFER, THERMAL ENGINEERING and ENVIRONMENT (HTE11), Florence, Italy, 2011*, Comparing Solutions of Hyperbolic and Parabolic Heat Conduction Equations for L-shape Samples, WSEAS Press, 2011, 384 – 389.

[6] S. Blomkalna, M. Buike, A. Buikis. Article in proceedings. In: *Proceedings of the 9th IASME/WSEAS International Conference on HEAT TRANSFER, THERMAL ENGINEERING and ENVIRONMENT (HTE11), Florence, Italy, 2011*, Several Intensive Steel Quenching Models for Rectangular and Spherical Samples, WSEAS Press, 2011, 390 – 395.

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ECG PROCESSING ALGORITHMS IN MOBILE TRAINING SYSTEM

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With the increase of healthcare services in non-clinical environments the processing and analysis of wearable sensors are growing significantly. The ITEA2 project CareWare Electronic Wearable Sport and Health Solutions is dealing with this demand. The aim of this project is to develop and leverage novel unobtrusive cyber physical systems for monitoring and advancing personal health and wellbeing. The integrated solution of different sensors such as smart interfaces, modelling and data analysis techniques should warrant that the created system is comfortable and effective.

Health condition is usually carried out stationary. However, this technique is inconvenient for professionals or ordinary people. Furthermore, there are some difficulties in evaluating health changes during physical activity. Each person has its own individual characteristics and needs optimal training mode. Otherwise, if the physical activity is too intense, injuries or overtraining may appear. One of the biggest disorders which might appear during training is heart problems. The electrodes integrated into T-shirts allow the registration of the ECG (electrocardiogram) signals. However, these signals contain low and high frequency noise. This paper contains the ECG filtering analysis and extracting of important for evaluation of health status parameters of ECG.

Each heartbeat is an electrical impulse (wave) which travels through heart [1]. In most cases, ECG signals are taken in stationary conditions to reduce the noise. In medicine and diagnostics, the ECG signal must be free from noise and undesired disturbance. Even though the ECG has its own variability the noise appears in the signal from many sources such as electrode contact noise, power line interfaces, respiration or instrumentation noise generated by electronic devices etc. [2]. In this paper the ECG signals are analysed when participant is walking and doing squats. Also, electrodes are close to skin because they are embedded into T-shirts to reduce the noise. The noisy ECG data can be modelled as in the equation (1)

$$y(x) = f(x) + v(x) + w(x), x = 0, ..., N \tag{1}$$

where f is a low-pass signal, w is a stationary white Gaussian noise and v is a sparse-derivative signal [3]. There are three main problems in initial ECG signal data analysis: ECG signal trend removal, Noise reduction, ECG parameters calculation and data compression for storage.

REFERENCES

[1] M. B. Velasco, B. Weng, K. E. Barner. ECG Signal Denoising and Baseline Wander Correction Based on the Empirical Mode Decomposition. *Science direct journal. Computers in Biology and Medicine*, **38** 1–13, 2008.

[2] N. Akshay, N. A. V. Jonnabhotla, N. Sadam, N. D. Yeddanapudi. ECG Noise Removal and QRS Complex Detection Using UWT. *ICEIE*, **V2** 438–442, 2010.

[3] I. W. Selesnick, H. L. Graber, D. S. Pfeil and R. L. Barbour. Simultananeous Low-pass Filtering and Total Variation Denoising. *IEEE Trans. Signal Process*, **62** (5):1109–1124, 2014.

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GENERALIZED GYROTRON MULTIMODE TIME-DEPENDENT GYROTRON EQUATIONS

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Until now mode competition in gyrotrons were investigated on the supposition that the transit time of electrons through the interaction space is much shorter than the cavity decay time. This assumption is valid for short and/or high diffraction quality resonators. However, in the case of long and/or low diffraction quality resonators, which are often utilized, this assumption is no longer valid. In such a case a different mathematical formalism has to be used for studying nonstationary oscillations. In [1] we developed the new formalism for the case of single mode oscillations. In the present paper we extend the formalism to the multimode case and present some results of mode competition calculations obtained by means of the generalized algorithm.

REFERENCES

[1] O. Dumbrajs and H. Kalis . P. *h*, **y** s.ics of plasmas 22, 053113 (2015). Nonostationary oscillations in gyrotrons revisited Physics of plasmas 22 053113-1–053113-6, 2015

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DIFFERENTIAL TRANSFORM METHOD FOR SOLVING A BOUNDARY VALUE PROBLEM ARISING IN CHEMICAL REACTOR THEORY

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In this study, we deal with the numerical solution of the mathematical model for an adiabatic tubular chemical reactor which processes an irreversible exothermic chemical reaction. For steady state solutions, the model can be reduced to the following nonlinear ordinary differential equation [1]:

$$\mathsf{u}'' - \lambda \mathsf{u}' + \lambda\mu(\beta - \mathsf{u})\mathsf{exp}(\mathsf{u}) = 0, \tag{1}$$

where λ, μ, and β are Péclet number, Damkohler number, and adiabatic temperature rise, respectively.
Boundary conditions of the equation (1) are

$$\mathsf{u}'(0) = \lambda \mathsf{u}(0), \mathsf{u}'(1) = 0. \tag{2}$$

Differential transform method [2] is used to solve the problem (1)-(2) for some values of the considered parameters. Residual error computation is adopted to confirm the accuracy of the results. In addition, the obtained results are compared with those obtained by other existing numerical approach [3].

REFERENCES

[1] R. Heinemann and A.B. Poore. Multiplicity, Stability, and Oscillatory Dynamics of the Tubular Reactor. *Chemical Engineering Science*, **36** (8): 1411–1419, 1981.

[2] V. S. Erturk and S. Momani. A reliable algorithm for solving tenth-order boundary value problems. *Numerical Algorithms*, **44** (2): 147–158, 2007.

[3] H. Q. Kafri, S. A. Khuri and A. Sayfy . A Fixed-Point Iteration Approach for Solving a BVP Arising in Chemical Reactor Theory. *Chemical Engineering Communications*, **204** (2): 198–204, 2017.

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NONCLASSICAL INITIAL-BOUNDARY VALUE PROBLEMS: THEIR REFORMULATION AND NUMERICAL SOLUTION

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Several physical phenomena are modeled by nonclassical initial-boundary value problems in one space variable, problems which involve an integral over the spatial domain of a function of the desired solution. This integral may appear in the boundary conditions and/or the governing partial differential equation. In this talk,we present several examples of problems of this type which arise in such diverse areas as chemical diffusion, heat conduction, thermoelasticity, population dynamics, vibration problems, nuclear reactor dynamics, and certain biological processes. It is shown how the example problems can be converted to a form to which standard numerical techniques may be applied in a relatively straightforward manner.

Special attention is devoted to a nonclassical problem which has received much attention in the literature, namely the diffusion equation subject to the specification of mass. For its numerical solution, we describe methods involving orthogonal spline collocation for the spatial discretization and the Crank-Nicolson method for the time stepping [1; 2; 3]. Also, we propose a finite-difference method based on Keller's box scheme; cf. [4].

REFERENCES

[1] B. Bialecki and G. Fairweather. Orthogonal spline collocation methods for partial differential equations. *J. Comput. Appl. Math.*, **128,** 55–82, 2001.

[2] B. Bialecki, G. Fairweather and J. C. López-Marcos. The Crank-Nicolson Hermite cubic orthogonal spline collocation method for the heat equation with nonlocal boundary conditions. *Adv. Appl. Math. Mech.*, **5,** 442–460, 2013.

[3] B. Bialecki, G. Fairweather and J. C. López-Marcos. The extrapolated Crank-Nicolson orthogonal spline collocation method for a quasilinear parabolic problem with nonlocal boundary conditions. *J. Sci. Comput.*, **62,** 265–283, 2015.

[4] G. Fairweather and R. D. Saylor. The reformulation and numerical solution of certain non–classical initial–boundary value problems. *SIAM J. Sci. Statist. Comput.*, **12,** 127–144, 1991.

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MIXED FINITE ELEMENT METHOD FOR THERMALLY COUPLED NONLINEAR DARCY FLOWS

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We consider thermally coupled nonlinear Darcy flows where the velocity and the pressure obey power law. More precisely, the coupled model problem is as follows:

$$\begin{cases} -div(k(\theta)|\nabla u|^{p-2}\nabla u) = f & \text{in } \Omega, \\ -\Delta\theta = k(\theta)|\nabla u|^{p} & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma, \\ \theta = 0 & \text{on } \Gamma, \end{cases} \tag{1}$$

where Ω is a bounded convex polygonal domain, Γ is its boundary, and $1 < p < \infty$. The pressure of the fluid is denoted by u, while θ denotes the temperature, and $k(\theta)$ is the viscosity dependent on the temperature. The problem (1) is an important model in studying the non-Newtonian flows with thermal effects (see, e.g., [1]).

Recently, Zhu et al. [3] have proposed and analyzed a mixed formulation of problem (1) where the velocity $\sigma = k(\theta)|\nabla u|^{p-2}\nabla u$ of the fluid is introduced as a new variable. One of the advantages of a such formulation is to get a more precise numerical solution for the velocity σ. However, the mixed formulation studied in [3] does not introduce the heat flux as a new unknown which means that the precision on the numerical solution of the heat flux is less than the one of the velocity σ.

The main purpose of this work is to study a mixed formulation of problem (1) where the velocity σ of the fluid and the heat flux $\xi = \nabla\theta$ are introduced as new unknowns. Based on such a formulation, a dual mixed finite element is constructed and analyzed. In this method, the lowest degree Raviart-Thomas finite element [2] is used for the approximation of σ and ξ. This finite element method enables us to obtain precise approximations of the dual variables which are the velocity σ and the heat flux ξ. Furthermore, it has local conservation properties analogous to the finite volume methods.

REFERENCES

[1] R. P. Gilbert and M. Fang. Nonlinear systems arising from nonisothermal, non-Newtonian Hele-Shaw flows in the presence of body forces and sources. *Math. Comput. Modelling*, **35** 1425–1444, 2002.

[2] P. A. Raviart and J. M. Thomas. A mixed finite element method for second order elliptic problems. *In Mathematical Aspects of the Finite Element Method, Lecture Notes in Math., Springer-Verlag, Berlin*, **606** 292–315, 1977.

[3] Jiang Zhu, Jiansong Zhang, Abimael F. D. Loula and Luiz Bevilacqua. Mixed variational formulation and numerical analysis of thermally coupled nonlinear Darcy flows. *SIAM J. Numer. Anal.*, **51** (5): 2746–2772, 2013.

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HIGH FREQUENCY ELECTRICAL OSCILLATIONS IN CAVITIES

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If the interior of a conducting cavity (such as a capacitor or a coaxial cable) is supplied with a very high-frequency electric signal, the information between the walls propagates with an appreciable delay, due to the finiteness of the speed of light. The configuration is typical of cavities having size larger than the wavelength of the injected signal. Such a non rare situation, in practice, may cause a break down of the performances of the device. We show that the classical Coulomb's law and/or Maxwell's equations do not correctly predict this behavior. Therefore, we provide an extension of the modeling equations (see also [1]) that allows for a more reliable determination of the electromagnetic field during the evolution process.

The main issue is that, even in vacuum (no dielectric inside the device), the fast variation of the signal produces sinks and sources in the electric field, giving rise to areas where the divergence is not zero. These regions are well balanced, so that their average in the domain is zero. However, this behavior escapes the usual treatment with classical electromagnetism. After recalling the main motivations and the theoretical results presented in [1], where this problem has been initially investigated, further examples and numerical experiment will be discussed.

REFERENCES

[1] D. Funaro. *Electromagnetism and the Structure of Matter*. World Scientific, Singapore, 2008.

[2] D. Funaro. High-frequency fields in capacitors. *Reports on Mathematical Physics*, 2017.

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FINITE ELEMENT ANALYSIS OF COUPLED HEAT TRANSFER AND WATER VAPOR EXCHANGE IN 3D TEXTILE LAYER

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Polypropylene can be applied in different market sectors (household goods, automotive industry, fibers). Due to the comparatively higher brittle temperature of polypropylene, its use in low-temperature environments should be carefully considered [1].

We present a computational model developed for investigation of heat and vapor mass exchange between the human body and environment through complex spatial through-thickness structure of 3D textile made of polypropylene. The practical need of such calculation is dictated by the necessity to find the effective heat transfer, heat capacity, water vapor transmission and air permeability coecients, as well as water vapor capacity of modern textile materials [2] in order to predict their functional properties.

In this work, the system of differential equations was discretized by finite element method. The obtained 3D model was solved by using COMSOL Multiphysics software. The fully coupled Multiphysics analysis was carried out. The Heat Transfer in Porous Media interface was used for modelling the heat transfer by conduction and convection. The Free and Porous Media Flow interface was used to compute fluid velocity and pressure fields of single-phase flow where free flow was coupled to porous media flow. In this study type stationary analysis of the system was performed [3].

REFERENCES

[1] D. Tripathi. *Practical Guide to Polypropylene*. Rapra Technology Limited, UK, 2002.

[2] R. Barauskas, A. Abraitiene. A model for numerical simulation of heat and water vapor exchange in multi-layer textile packages with three-dimensional spacer fabric ventilation layer. *Textile Research Journal*, **DOI:** 10.1177/0040517510392468, 2011.

[3] COMSOL MULTIPHYSICS 5.2a . https://www.comsol.com/products. *2017*, 03.

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ON POLYNOMIAL DYNAMICAL SYSTEMS AND THEIR APPLICATIONS

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We carry out the global bifurcation analysis of low-dimensional polynomial dynamical systems. To control all of their limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all of their rotation parameters. It can be done by means of the development of new bifurcational geometric methods based on Perko's planar termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity) [1]. This principle is a consequence of the principle of natural termination which was stated for higher-dimensional dynamical systems by A. Wintner who studied one-parameter families of periodic orbits of the restricted three-body problem and used Puiseux series to show that in the analytic case any one-parameter family of periodic orbits can be uniquely continued through any bifurcation except a period-doubling bifurcation. Such a bifurcation can happen, e. g., in a three-dimensional Lorenz system. But this cannot happen for planar systems. That is why the Wintner–Perko termination principle is applied for studying multiple limit cycle bifurcations of planar polynomial dynamical systems [1]. If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Applying this approach, we have solved, e. g., Smale's Thirteenth Problem for the classical Liénard system [2]. Generalizing the obtained results, we have solved also the Problem on the maximum number and distribution of limit cycles for the general Liénard polynomial system with an arbitrary number of singular points [3]. We have also applied this approach for studying global limit cycle bifurcations of Holling-type systems which model the population dynamics in biomedical and ecological systems [4] Finally, applying a similar approach, we have considered various applications of three-dimensional polynomial dynamical systems and, in particular, completed the strange attractor bifurcation scenario in the classical Lorenz system globally connecting the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles [5].

REFERENCES

[1] V.A. Gaiko. *Global bifurcation theory and Hilbert's sixteenth problem.* Kluwer Academic Publishers, Boston, 2003.

[2] V.A. Gaiko. On limit cycles surrounding a singular point. *Differ. Equ. Dyn. Syst.*, **20** (3):329–337, 2012.

[3] V.A. Gaiko. Maximum number and distribution of limit cycles in the general Liénard polynomial system. *Adv. Dyn. Syst. Appl.*, **10** (2):177–188, 2015.

[4] V.A. Gaiko. Global qualitative analysis of a Holling-type system. *Int. J. Dyn. Syst. Differ. Equ.*, **6** (2):161–172, 2016.

[5] V.A. Gaiko. Global bifurcation analysis of the Lorenz system. *J. Nonlinear Sci. Appl.*, **7** (6):429–434, 2014.

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ZEROS OF SOME COMBINATIONS OF DIRICHLET L-FUNCTIONS AND HURWITZ ZETA-FUNCTIONS

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Let χ be a Dirichlet character and α, $0 < \alpha \leq 1$, be a fixed parameter. The Dirichlet L-function $L(s,\chi)$ and Hurwitz zeta-function $\zeta(s,\alpha)$, $s = \sigma + it$, are defined, for $\sigma > 1$, by the series

$$L(s,\chi) = \sum_{m=1}^{\infty} \frac{\chi(m)}{m^s} \qquad \text{and} \qquad \zeta(s,\alpha) = \sum_{m=0}^{\infty} \frac{1}{(m+\alpha)^s},$$

and are continued meromorphically to the whole complex plane. We consider the number of zeros of the functions $F\left(L(s,\chi_1),\dots,L(s,\chi_r)\right)$, $F_1\left(\zeta(s,\alpha_1),\dots,\zeta(s,\alpha_k)\right)$ and $F_2(L(s,\chi_1),\dots,L(s,\chi_r),\zeta(s,\alpha_1),\dots,\zeta(s,\alpha_k))$ for some classes of characters $\chi_1,\dots,\chi_r$, parameters $\alpha_1,\dots,\alpha_k$, and operators F, F_1, and F_2. For example, the following theorem is valid. Let $D = \left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$ and let $H(D)$ denote the space of analytic functions on D endowed with the topology of uniform convergence on compacta. Moreover, let $S = \{g \in H(D) : g(s) \neq 0 \quad \text{or} \quad g(s) \equiv 0\}$.

THEOREM 1. *Suppose that $F : H^{r+k}(D) \to H(D)$ is a continuous operator such that, for every polynomial $p = p(s)$, the set $(F^{-1}\{p\}) \cap (S^r \times H^k(D))$ is non-empty, $\chi_1,\dots,\chi_r$ are pairwise non-equivalent Dirichlet characters, and the set*

$$\{(\log p : p \text{ is prime}), (\log(m+\alpha_j) : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \quad j = 1,\dots,k)\}$$

is linear independent over the field of rational numbers $\mathbb{Q}$. Then, for every σ_1, σ_2, $\frac{1}{2} < \sigma_1 < \sigma_2 < 1$, there exists a constant $c = c(\chi_1,\dots,\chi_r,\alpha_1,\dots,\alpha_k,F,\sigma_1,\sigma_2) > 0$ such that, for sufficiently large T, the function

$$F\left(L(s,\chi_1),\dots,L(s,\chi_r),\zeta(s,\alpha_1),\dots,\zeta(s,\alpha_k)\right)$$

has more than cT zeros lying in the rectangle $\{s \in \mathbb{C} : \sigma_1 < \sigma < \sigma_2,\ 0 < t < T\}$.

For example, if χ_1 and χ_2 are non-equivalent characters, and the numbers α_1 and α_2 are algebraically independent over $\mathbb{Q}$, then the function $L(s,\chi_1)L(s,\chi_2)\zeta(s,\alpha_1)\zeta(s,\alpha_2)$ satisfies the hypotheses of Theorem 1.

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BOUNDARY VALUE PROBLEM FOR THE THIRD ORDER ASYMPTOTICALLY LINEAR SYSTEM

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We consider the third order system $\boldsymbol{x}''' = \boldsymbol{f}(\boldsymbol{x})$ with the boundary conditions $\boldsymbol{x}(0) = \mathbf{0}$, $\boldsymbol{x}(1) = \mathbf{0}$, $\boldsymbol{x}'(1) = \mathbf{0}$, where the vector field $\boldsymbol{f} \in C^1(\mathbb{R}^n, \mathbb{R}^n)$ is asymptotically linear and $\boldsymbol{f}(\mathbf{0}) = \mathbf{0}$. We provide the existence results using the vector field rotation theory.

REFERENCES

[1] A. Gritsans, F. Sadyrbaev and I. Yermachenko. Dirichlet boundary value problem for the second order asymptotically linear system. *International Journal of Differential Equations*, **2016** (Article ID 5676217): 12 pages, 2016.

[2] M.A. Krasnosel'skiĭ and P.P. Zabreĭko. *Geometrical Methods of Nonlinear Analysis*. Springer, Berlin-New York, 1984.

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APPROXIMATION OF THE SET OF TRAJECTORIES OF THE CONTROL SYSTEM WITH LIMITED CONTROL RESOURCES

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Integral equations arise in different problems of theory and applications. In this presentation the control system described by the Urysohn integral equation

$$x(\xi) = f(\xi, x(\xi)) + \lambda \int_E \left[K_1(\xi, s, x(s)) + K_2(\xi, s, x(s))u(s)\right] ds \tag{1}$$

is considered, where $x(s) \in \mathbb{R}^n$ is the state vector, $u(s) \in \mathbb{R}^m$ is the control vector, $(\xi, s) \in E \times E$, $E \subset \mathbb{R}^k$ is a compact set, $\lambda \in \mathbb{R}$.

For given $p > 1$ and $r > 0$ the set $U_{p,r} = \left\{u(\cdot) \in L_p\left(E; \mathbb{R}^m\right) : \left\|u(\cdot)\right\|_p \leq r\right\}$ is called the set of admissible control functions and each $u(\cdot) \in U_{p,r}$ is said to be an admissible control function, where $L_p\left(E; \mathbb{R}^m\right)$ is the space of Lebesgue measurable functions $u(\cdot) : E \to \mathbb{R}^m$ such that $\left\|u(\cdot)\right\|_p < +\infty$, $\left\|u(\cdot)\right\|_p = \left(\int_E \left\|u(s)\right\|^p ds\right)^{\frac{1}{p}}$, $\|\cdot\|$ denotes the Euclidean norm. It is assumed that

A. The functions $f(\cdot) : E \times \mathbb{R}^n \to \mathbb{R}^n$, $K_1(\cdot) : E \times E \times \mathbb{R}^n \to \mathbb{R}^n$ and $K_2(\cdot) : E \times E \times \mathbb{R}^n \to \mathbb{R}^{n \times m}$ are continuous;

B. There exist $l_0 \in [0, 1)$, $l_1 > 0$ and $l_2 > 0$ such that $\left\|f(\xi, x_2) - f(\xi, x_1)\right\| \leq l_0 \left\|x_2 - x_1\right\|$ and

$$\left\|K_1(\xi, s, x_2) - K_1(\xi, s, x_1)\right\| \leq l_1 \left\|x_2 - x_1\right\|, \ \left\|K_2(\xi, s, x_2) - K_2(\xi, s, x_1)\right\| \leq l_2 \left\|x_2 - x_1\right\|$$

for every $(\xi, s, x_1) \in E \times E \times \mathbb{R}^n$ and $(\xi, s, x_2) \in E \times E \times \mathbb{R}^n$;

C. The inequality $0 \leq \lambda \left[l_1 \mu\left(E\right) + l_2 \left[\mu(E)\right]^{\frac{p-1}{p}} r\right] < 1 - l_0$ is satisfied, where $\mu(E)$ denotes the Lebesgue measure of the set E.

Let $u(\cdot) \in U_{p,r}$. A continuous function $x(\cdot) : E \to \mathbb{R}^n$ satisfying the equation (1) for every $\xi \in E$ is said to be a trajectory of the system (1), generated by the admissible control function $u(\cdot)$. The set of all trajectories of the system (1) generated by all admissible control functions $u(\cdot) \in U_{p,r}$ is denoted by $\mathbf{X}_{p,r}$.

Approximation of the set of trajectories $\mathbf{X}_{p,r}$ is studied. Step by step way, the set of control functions is replaced by the set consisting of a finite number of control functions which generate the finite number of trajectories. An upper bound evaluation for the Hausdorff distance between the set $\mathbf{X}_{p,r}$ and the set consisting of a finite number of trajectories is obtained.

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ON SELF-REGULARIZATION OF ILL-POSED PROBLEMS IN BANACH SPACES BY LEAST SQUARES AND LEAST ERROR METHOD

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We consider an ill-posed problem $Au = f$ with linear operator $A \in \mathcal{L}(E, F)$ acting between Banach spaces E, F. The exact right-hand side f is unknown; instead noisy data f^δ satisfying $\|f^\delta - f\| \le \delta$ with known noise level δ are given. For finding approximation u_n to the solution u_* of the problem we use projection methods (see [2]), [3]). In the least squares method we use finite dimensional subspaces $E_n \subseteq E$ with properties $E_n \subseteq E_{n+1}$ $\forall n \in \mathbb{N}$, $\overline{\bigcup_{n\in\mathbb{N}} E_n} = E$ and take for the approximate solution

$$u_n \in \operatorname{argmin}\{\|A\tilde{u}_n - f^\delta\| : \tilde{u}_n \in E_n\}.$$

In the least error method we use finite dimensional subspaces $Z_n \subseteq F^*$ with properties $Z_n \subseteq Z_{n+1}$ $\forall n \in \mathbb{N}$, $\overline{\bigcup_{n\in\mathbb{N}} Z_n} = F^*$ and take for the approximate solution

$$u_n \in \operatorname{argmin}\{\|\tilde{u}\| : \tilde{u} \in E, \quad \langle z_n, A\tilde{u}\rangle = \langle z_n, f^\delta\rangle \quad \forall z_n \in Z_n\}. \tag{1}$$

For the case of exact data ($\delta = 0$) we give conditions under which the discretized problem has a unique solution u_n and the convergence $\|u_n - u_*\| \to 0 \quad (n \to \infty)$ to the exact solution u_* is guaranteed. If these conditions hold but the data are noisy ($\delta > 0$), then one can choose the dimension $n = n(\delta)$ of the projection space as the regularization parameter depending on the noise level δ in such way that $\|u_{n(\delta)} - u_*\| \to 0$ as $\delta \to 0$. Such regularization by discretization is called self-regularization. For the choice of dimension $n = n(\delta)$ we consider a priori rule and a posteriori choice by the discrepancy principle and by the monotone error rule (see [2]).

Note that self-regularization of ill-posed problems by projection methods in Hilbert spaces was studied in [1], [4].

REFERENCES

[1] A. Ganina, U. Hämarik and U. Kangro . On the self-regularization of ill-posed problems by the least error projection method. *Mathematical Modelling and Analysis*, **19** (3):299 – 308, 2014.

[2] U. Hämarik, B. Kaltenbacher, U. Kangro and E. Resmerita. Regularization by Discretization in Banach Spaces. *Inverse Problems*, **32** (3):035004 (28pp), 2016.

[3] F. Natterer. Regularisierung schlecht gestellter probleme durch Projektionsverfahren. *Numer. Math.*, **28** (3):329 – 341, 1977.

[4] G. Vainikko and U. Hämarik. Projection methods and self-regularization in ill-posed problems. *Izv. Vyssh. Ucebn. Zaved. Mat.*, **15** (10):3 – 17, 1985.

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STABILITY OF TWO–DIMENSIONAL PARABOLIC EQUATION WITH AN INTEGRAL BOUNDARY CONDITION

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We consider finite difference method for a class of two–dimensional parabolic equations with integral boundary condition, like equation (1)

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(x,y)\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(x,y)\frac{\partial u}{\partial y}\right) - q(x,y)\cdot u + f(x,y,t), \quad (x,y)\in\Omega, \tag{1}$$

with integral condition on boundary

$$u(x,y,t) = \iint\limits_{\Omega} K(x,y,\xi,\eta)d\xi d\eta + \mu(x,y,t), \quad (x,y)\in\partial\Omega \tag{2}$$

and initial condition

$$u(x,y,0) = \varphi(x,y), \quad (x,y)\in\Omega. \tag{3}$$

This is a specific problem, since in the nonlocal condition the values of solution at contour points are associated with the double integral in the whole domain. The stability of difference scheme is proven using the properties of the M–matrices. The numerical results of some examples are presented, approving our theoretical investigations.

REFERENCES

[1] Y. Lin, S. Xu, H. M. Yin. Finite difference approximations for a class of non–local parabolic equations. *Intern. J. Math. and Mat. Sci.*, **20** (1):147–164, 1997.

[2] K. Jakubėlienė and M. Sapagovas . On the stability of a difference scheme for a two–dimensional parabolic equation with an integral condition. *Lith. Math. J.*, **53** (3):311–323, 2013.

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IDENTIFICATION OF A KERNEL IN AN EVOLUTIONARY INTEGRAL EQUATION

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We consider an inverse problem to determine a kernel in an evolutionary integral equation occurring in modelling of subdiffusion. We prove the existence, uniqueness and stability of a solution of the inverse problem in an abstract setting. Results are global in time.

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ON SOME APPLICATIONS OF THE LAMBERT W FUNCTION AND THE POLYLOGARITHM FUNCTION IN PROBABILITY AND STATISTICS

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The lecture discusses the increasing interest in the use of some special functions in probability and statistics. Specifically, the attention will be focused on the Lambert W function (see [2] for a survey) and the polylogarithm function (see [8] and the references therein), also highlighting the role of the Lerch transcendent function ([1]). These functions became popular with their implementation in computer algebra systems and programming languages. In spite of their long history in mathematics and physics, in the fields of applied probability and statistics the aforementioned functions have been used mainly only during the last decade. After an overview of the history and basic concepts of these functions, the lecture describes recent applications which are related to well-known probability distributions such as: Gomperz–Makeham, Erlang, half-logistic, Lindley, Log-Lindley, Muth, Poisson–Lindley and Weibull-geometric, among others (see [3; 4; 5; 6; 7]).

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REFERENCES

[1] T.M. Apostol. Zeta and related functions. In: *NIST Handbook of Mathematical Functions*, F.W.F. Olver, D.W. Lozier, R.F. Boisvert, C.W. Clark (Eds.), National Institute of Standards and Technology, Washington, DC, and Cambridge University Press, 2010, 601–616.

[2] R.M. Corless, G.H. Gonnet, D.E.G. Hare, D.J. Jeffrey and D.E. Knuth. On the Lambert W function. *Adv. Comput. Math.*, **5** (4): 329–359, 1996.

[3] P. Jodrá. On a connection between the polylogarithm function and the Bass diffusion model. *Proc. R. Soc. A*, **464** (2099):3081–3088, 2008.

[4] P. Jodrá. Computer generation of random variables with Lindley or Poisson-Lindley distribution via the Lambert W function. *Math. Comput. Simulation*, **81** (4): 851–859, 2010.

[5] P. Jodrá and M.D. Jiménez-Gamero. On a logarithmic integral and the moments of order statistics from the Weibull-geometric and half-logistic families of distributions. *J. Math. Anal. Appl.*, **410** (2): 882–890, 2014.

[6] P. Jodrá and M.D. Jiménez-Gamero. A note on the Log-Lindley distribution. *Insurance Math. Econom.*, **71**: 189–194, 2016.

[7] P. Jodrá, M.D. Jiménez-Gamero and M.V. Alba-Fernández. On the Muth distribution. *Math. Model. Anal.*, **20** (3): 291–310, 2015.

[8] L.C. Maximon. The dilogarithm function for complex argument. *Proc. R. Soc. A*, **459** (2039):2807–2819, 2003.

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CLASS OF ZETA-FUNCTIONS: REMARKS ON THE MIXED JOINT UNIVERSALITY

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The first result on the investigation of so called "mixed" joint universality for a collection of different type zeta-functions (having the Euler product expansion and without it) is due to H. Mishou. In 2007, he proved [3] that the pair of zeta-functions consisting of the Riemann zeta-function $\zeta(s)$ and the Hurwitz zeta-function $\zeta(s,\alpha)$ is universal in Voronin sense.

For $m \in \mathbb{N}$, attach $g(m) \in \mathbb{N}$, and, for $j \in \mathbb{N}$ with $1 \le j \le g(m)$, let $f(j,m) \in \mathbb{N}$ and $a_m^{(j)} \in \mathbb{C}$. Denote by p_m the mth prime number, and let $s = \sigma + it$ be a complex variable. The zeta-function $\varphi(s)$ is defined by the polynomial Euler product $\varphi(s) = \prod_{m=1}^{\infty} A_m^{-1}(p_m^{-s})$ with polynomials $A_m(x) := \prod_{j=1}^{g(m)}(1 - a_m^{(j)} x^{f(j,m)})$. Suppose that $g(m) \le cp_m^{\alpha}$, $|a_m^{(j)}| \le p_m^{\beta}$ with $c > 0$, and some non-negative constants α and β. The infinite product converges absolutely for $\sigma > \alpha + \beta + 1$.

Let $\mathfrak{B} = \{b_m : m \in \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers with minimal period $l \in \mathbb{N}$, and let $\gamma \in \mathbb{R}$, $0 < \gamma \le 1$, be a fixed parameter. Then the function $\zeta(s,\gamma;\mathfrak{B})$ is defined, for $\sigma > 1$, by the series $\zeta(s,\gamma;\mathfrak{B}) = \sum_{m=0}^{\infty} \frac{b_m}{(m+\gamma)^s}$. From the periodicity of $\mathfrak{B}$ we have $\zeta(s,\gamma;\mathfrak{B}) = \frac{1}{l^s}\sum_{k=0}^{l-1} b_k \zeta(s,(k+\gamma)/l)$, $\sigma > 1$, where $\zeta(s,\gamma)$ is the classical Hurwitz zeta-function. Therefore, the function $\zeta(s,\gamma;\mathfrak{B})$ is a linear combination of the functions $\zeta(s,\gamma)$, and last equality gives analytic continuation for $\zeta(s,\gamma;\mathfrak{B})$ to the whole complex plane, where it is regular, except, maybe, for a simple pole at $s = 1$ with residue $b := \frac{1}{l}\sum_{k=0}^{l-1} b_k$.

The mixed joint universality theorem for the zeta-function $\varphi(s)$ belonging to the Steuding class $\tilde{S}$ and periodic Hurwitz zeta-function $\zeta(s,\gamma;\mathfrak{B})$ was obtained by the author and Kohji Matsumoto in [2]. In the talk, we will present the mixed joint universality for a class of zeta-functions, consisting of the so-called Matsumoto zeta-functions and tuple of periodic Hurwitz zeta-functions, when certain conditions are fulfilled. This result was proved in [1].

REFERENCES

[1] R. Kačinskaitė and K. Matsumoto. Remarks on the mixed joint universality for a class of zeta functions. *Bull. Aust. Math. Soc.*, **95** : 187-198, 2017.

[2] R. Kačinskaitė and K. Matsumoto. The mixed joint universality for a class of zeta-functions. *Math. Nachr.*, **288** : 1900–1909, 2015.

[3] H. Mishou. The joint value distribution of the Riemann zeta function and Hurwitz zeta-function. *Lith. Math. J.*, **47** : 32–47, 2007.

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ON MATHEMATICAL MODELLING OF THE SOLID-LIQUID MIXTURES TRANSPORT IN POROUS AXIAL-SYMMETRICAL CONTAINER WITH HENRY AND LANGMUIR SORPTION KINETICS

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The present talk considers the diffusion and convection filtration problem in axial-symmetrical container $\Omega = \{(r, z, \phi) : 0 \le r \le R, 0 \le z \le L, 0 \le \phi \le 2\pi\}$.
Domain Ω consist of porous material where through the pores of filter moves incompresible liquid - pollutants in z-direction.
For adsorption kinetics we use linear Henry's [1] and nonlinear Langmuir's [2] sorptions isotherms. We derive the convective-dispersion and nonequilibrium sorption equations in the following form [3]: $\frac{1}{r}\left(\frac{\partial}{\partial r}(D_r r\frac{\partial u}{\partial r})\right) + D_z\frac{\partial^2 u}{\partial z^2} + V_0\frac{\partial u}{\partial z} = m\frac{\partial u}{\partial t} + \frac{\partial a}{\partial t}$, $\frac{\partial a}{\partial t} = \beta(u - \tilde{u}), r \in [0, R], z \in [0, L] t > 0$,
where $a(r, z, t), u(r, z, t)$ are the concentrations in the adsorbed and aqueous phases for the pollutants, $a = \frac{\tilde{u}}{\gamma}$ is the expression for linear isotherm of Henry, D_r, D_z are diffusion coefficients , V_0=const is the pore water velocity in z-direction, m is the fraction of the total volume of the material occupied by pores, $\tilde{u}$ is concentration of pollutants, which is in local equilibrium conditions $\frac{\partial a}{\partial t} = 0$, t is the time, β is the sorption rate constant, $1/\gamma$ is Henry's coefficient for the sorbent characteristic. For nonlinear sorbtion $a = \frac{\tilde{u}}{\gamma(1+p\tilde{u}}$ (Langmuir's isotherm), where p is positive parameter (for $p = 0$ we have Henry's isotherm). For $t = 0$ we use homogeneous initial conditions $u(r, z, 0) = 0, a(r, z, 0) = 0$.
We use following boundary conditions: $\frac{\partial u(0,z,t)}{\partial r} = \frac{\partial a(0,z,t)}{\partial r} = 0, u(R, z, t) = a(R, z, t) = 0$,
$u(r, L, t) = u_0(t) = U_0(1 - \tanh(\alpha t))$, $\frac{\partial u(r,0,t)}{\partial z} = 0$, $\frac{\partial a(r,0,t)}{\partial z} = 0$,
$\alpha = const > 1, U_0 = const > 0$. The concentration u on the inlet is depending on t.
The approximation of corresponding initial boundary value problem of the system of PDEs is based on the conservative averaging method [1].

REFERENCES

[1] William Henry. Experiments on the quantity of gases absorbed by water. *Philsophical Trans.of the Royal Soc.*, **93** 29-274,1803.

[2] Irvin Langmuir. The adsorption of gases on plane surfaces of glass, mica and platinum. *J. Am. Chem.Soc.* , **40** 1361-1403, 1918.

[3] A.N.Tihonov, A.A. Samarsky. Equations of Mathematical Physics *Press " Nauka", Moskow,1966 (in Russian).* , .

[1]This work was partially supported by the grant 623/2014 of the Latvian Council of Science and ERAF project Nr.1.1.1.1/16/A/004

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ON PROJECTION METHODS FOR ILL-POSED PROBLEMS IN BANACH SPACES WHEN THE PROJECTORS ARE NOT UNIFORMLY BOUNDED

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We consider an ill-posed problem $Au = f$ with linear operator $A \in \mathcal{L}(E, F)$ acting between Banach spaces E, F. The exact right-hand side f is unknown; instead noisy data f^δ satisfying $\|f^\delta - f\| \le \delta$ with known noise level δ are given. For finding approximation u_n to the solution u_* of the problem we use projection methods studied in [1] in Hilbert spaces and [2] in Banach spaces).

Let $E_n \subseteq E$, $Z_n \subseteq F^*$, $n \in \mathbb{N}$, be finite dimensional nontrivial subspaces. The approximation u_n is defined by

$$u_n \in E_n : \ \langle z_n, Au_n\rangle_{F,F^*} = \langle z_n, f^\delta\rangle \ \forall z_n \in Z_n.$$

In case of noisy data the choice of the dimension of the discretization space is essential to guarantee the convergence of the method as $\delta \to 0$. Convergence when n is chosen by the discrepancy principle, depending on the noise level, is considered in [2]. One of the conditions there was existence of $\tau > 0$ such that

$$\tau \sup_{z_n \in Z_n, \|z_n\|_{F^*}=1} \langle z_n, Aw_n\rangle_{F,F^*} \ge \|Aw_n\|_F \quad \forall w_n \in E_n, \ \forall n \ge N_0.$$

Then the discrepancy principle is to choose the first index $n = n(\delta)$ such that $\|Au_n - f^\delta\|_F \le b\delta$ with b fixed, $b > \tau + 1$.

For collocation methods for integral equations existence of τ means the uniform boundedness of the interpolation projector onto the subspace $AE_n \subset F$. If $F = C^m$ and E_n is the space of polynomials of degree not exceeding n, this is generally not true. We consider a modification of the discrepancy principle if instead of τ we have τ_n depending on n, which may be unbounded as $n \to \infty$. Then instead of fixed b we should use b_n depending on n. We prove the convergence of this modification.

We also present some numerical examples which show convergence of the algorithm as $\delta \to 0$.

REFERENCES

[1] G. Vainikko and U. Hämarik. Projection methods and self-regularization in ill-posed problems. *Izv. Vyssh. Ucebn. Zaved. Mat.*, **15** (10):3 – 17, 1985.

[2] U. Hämarik, B. Kaltenbacher, U. Kangro and E. Resmerita. Regularization by Discretization in Banach Spaces. *Inverse Problems*, **32** (3):035004 (28pp), 2016.

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DISCRIMINATION POWER OF KNOWLEDGE TESTING ITEMS. FUZZY MATHEMATICAL MODEL

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In the article [1] it was proposed to describe the features of knowledge testing items by non-decreasing functions $k_j(p) : [0,1] \to [0,1]$, when the independent variable p represents the student's knowledge level. Characteristic function of j-th item $k_j(p)$ is interpreted as the probability of a correct response to the j-th test item for students whose knowledge level p. Function's $k_j(p)$ derivative $k'_j(p)$ shows how the item j differentiates students according to their knowledge. For example, if $k'_1(0.2) = \max\limits_{p\in[0,1]} k'_1(0.2)$ and $k'_2(0.8) = \max\limits_{p\in[0,1]} k'_2(0.8)$, then the first item to better differentiate weak students, and the second – strong students. Such classification strongly depends not only on the items $k_j(p)$ functional form, but also requires to measure the students level of knowledge p which is difficult to do in practice, and methodologically.

In this study we proposed to apply fuzzy sets theory to describe knowledge test items. Subsets W, M, S – of respectively weak, average and strong students are constructed. For each subset the difficulty of test item is determined in the form of fuzzy triangle number $Tr(L,T,R)$, $(0 \leqslant L \leqslant T \leqslant R \leqslant 100)$ regarded as the percentage of students correctly responded to the item. For the triangle numbers partial strict $Tr_1 \prec Tr_2$ and non-strict $Tr_1 \preccurlyeq Tr_2$ order relationships are defined. The item is good for differentiating of all students if $Tr_W \prec Tr_M \prec Tr_S$; of weak students if $Tr_W \prec Tr_M \preccurlyeq Tr_S$; of strong students – $Tr_W \preccurlyeq Tr_M \prec Tr_S$. When the only relationship $Tr_W \preccurlyeq Tr_M \preccurlyeq Tr_S$ is valid then the item poorly differentiates students and finally, when it is none of the above four cases the item is considered as inappropriate.

Selection methodology of the subsets W, M, S and design algorithm of fuzzy triangles $Tr(L,T,R)$ are proposed. The features of functions $k_j(p)$ and fuzzy classification comparison strategies are discussed.

REFERENCES

[1] A. Krylovas and N. Kosareva.. Mathematical modelling of forecasting the results of knowlegdge testing. *Technological and Economic Development of Economy*, **14** (3):388–401, 2008.

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EDDY CURRENT TESTING OF MULTILAYER MEDIA WITH FLAWS

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Eddy current method is one of the widely used methods for quality testing of electrically conducting materials. It is based on the principle of electromagnetic induction. Eddy currents are induced in the medium if a coil carrying alternating current is located in the vicinity of the tested material. The change in impedance of the coil due to eddy currents can be used to detect anomalies (or flaws) in the conducting medium. Quasi-analytical method [1] for the solution of direct eddy current testing problems for multilayer media with cylindrical flaws is used to calculate the change in impedance of a cylindrical air core coil whose axis coincides with the axis of the cylindrical flaw. Such models can be used to assess the effect of corrosion (surface flaws) or test the quality of spot welding (volumetric flaws). The following examples are considered: (a) one or two cylindrical flaws in a plate or two-layer medium, and (b) a volumetric flaw in a half-space or plate. Calculations of the change in impedance with quasi-analytical method are in good agreement with finite element modeling.

REFERENCES

[1] T.P. Theodoulidis, and E.E. Kriezis. *Eddy current canonical problems (with applications to nondestructive evaluation)*. Tech Science Press, 2006.

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MATHEMATICAL MODELS FOR QUALITY TESTING OF ELECTRICALLY CONDUCTING MATERIALS

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Eddy current method is one of the widely used methods for quality testing of electrically conducting materials. If a source of alternating current (for example, a coil) is located in the vicinity of a conducting medium to be tested then eddy currents are induced in the medium due to the law of electromagnetic induction. These currents interact with the currents in the coil. The impedance of the coil changes as a result of such interaction (these changes can be measured experimentally).

In many applications of eddy current testing an inverse problem has to be solved (assessment of corrosion in metal plates, flaw detection in heat exchanger tubes, quality testing of spot welding). In order to solve the inverse problem an accurate and reliable method for the solution of the direct problem has to be used. In this talk we shall concentrate mainly on the solutions of direct problems. The starting point is the system of the Maxwell's equations for a homogeneous isotropic medium. Exact solutions of the corresponding problems for multilayer medium in Cartesian, cylindrical and spherical coordinates are briefly discussed. Analytical solutions are also possible for some cases where either electric conductivity or magnetic permeability depend on one spatial coordinate.

One of the most important applications of eddy current testing is related to flaw detection in a conducting medium. The presence of an inclusion destroys symmetry. However, axisymmetric problem can also be analyzed in case of special types of flaws. Perturbation methods are analyzed in an attempt to solve the direct problem in case a flaw is present in a conducting medium. There is one important shorcoming of perturbation methods - one cannot control the computational error.

One of the popular quasi-analytical methods for the solution of direct problems for flaw detection is the TREE method [1] where the abbreviation stands for "truncated eigenfunction expansions". The method is based on a simple physical assumption - the electromagnetic field is exactly zero at a sufficiently large distance b from the coil. The problem can be solved by separation of variables (the computational error can be controlled by choosing the value of b). The method is quasi-analytical since two steps of the procedure require the use of numerical methods: (a) calculation of complex eigenvalues without good initial guess and (b) solution of systems of linear equations. Examples of the application of the TREE method are considered.

REFERENCES

[1] T.P. Theodoulidis, and E.E. Kriezis. *Eddy current canonical problems (with applications to nondestructive evaluation)*. Tech Science Press, 2006.

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CLASSICAL SOLUTION OF THE MIXED PROBLEM FOR KLEIN-GORDON-FOCK EQUATION WITH NONLOCAL CONDITIONS

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One-dimensional Klein-Gordon-Fock equation

$$\partial_{tt}u - a^2\partial_{xx}u - \lambda(t,x)u = f(t,x), \tag{1}$$

is given on the domain $Q = (0,\infty)\times\mathbb{R}$.

Initial conditions

$$u(0,x) = \varphi(x), \partial_t u(0,x) = \psi(x), \quad x \in [0;l], \tag{2}$$

$l \in \mathbb{R}, l < +\infty$, and unlocal conditions

$$\begin{aligned} u(x_0,0) &= \int\limits_{\alpha_0^{(1)}(x_0)}^{\alpha_0^{(2)}(x_0)} K_0(x_0,s)u(x_0,s)ds + q_0(x_0), \\ u(x_0,l) &= \int\limits_{\alpha_l^{(1)}(x_0)}^{\alpha_l^{(2)}(x_0)} K_l(x_0,s)u(x_0,s)ds + q_l(x_0), \end{aligned} \tag{3}$$

where $\alpha_j^{(i)} : \mathbb{R}^+ \to \mathbb{R}, i = 1,2, j = 0,l$ and $0 \le \alpha_j^{(1)} \le \alpha_j^{(2)} \le l, j = 0,l$ are connected to the equation (1).

THEOREM 1. *Let* $f \in C^1(\overline{Q})$, $\lambda \in C^1(\overline{Q})$, $q_0 \in C^2([0;+\infty))$, $q_l \in C^2([0;+\infty))$, $\varphi \in C^2([0,l])$, $\psi \in C^1([0,l])$, $K_j(t) \in C^2\left([0,+\infty)\times[\alpha_j^{(1)}(t),\alpha_j^{(2)}(t)]\right)$ *and* $\alpha_j^{(i)} \in C^2([0,+\infty)), i = 1,2,\ j = 0,l$. *Classical solution of the problem (1) – (3) exists and is unique in class* $C^2(\overline{Q})$ *if and only if homogeneous matching conditions are fulfilled when* $k = 0$.

REFERENCES

[1] D.Gordeziani, G. Avalishvili. Investigation of the nonlocal initial-boundary value problems for the string oscillation and telegraph equation. *AMI*, **2** 65–79, 1997.

[2] V.Dmitriev. Unlocal problem with integral conditions for the wave equation. *SamSU proceedings - Natural science series*, **2** (42):15–26, 2006.

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COMPARATIVE ANALYSIS OF 3 MCDM METHODS EFFICIENCY

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Multiple criteria problem is discussed when m alternatives are compared according to performance values of alternatives evaluated in terms of n attributes (criteria). Decision making matrix is as follows: $R = \begin{pmatrix} r_1^{(1)} & r_2^{(1)} & \dots & r_n^{(1)} \\ r_1^{(2)} & r_2^{(2)} & \dots & r_n^{(2)} \\ \dots & \dots & \dots & \dots \\ r_1^{(m)} & r_2^{(m)} & \dots & r_n^{(m)} \end{pmatrix}$. The paper compares efficiency of three different multiple criteria decision methods. WEBIRA (WEight Balancing Indicator Ranks Accordance) approach includes methods for setting attributes weights by the so-called weights balancing procedure [1] when attributes naturally form 2 or 3 groups, for example, external and internal; qualitative and quantitative evaluations; economic, social and environmental aspects of sustainable development. In the first stage, attributes sequence is determined, for this purpose Kemeny median, entropy or voting theory methods are applied. In the second stage, the attribute weights are found by solving optimization problem. The weights are selected so that 2 groups of evaluation criteria (attributes) are best matched with each other, i. e. we maximize the set of the best alternatives according to both criteria. Monte Carlo experiments are conducted to compare WEBIRA, AVRG (simple average) and EMDCW (Entropy Method for Determining the Criterion Weight) [2] efficiency. 4 different methods of decision making matrix R initial values normalization are compared: Max, MinMax, Sum and Vector normalization. Random matrices are simulating repeated expert evaluations of the same alternatives. They were generated so, that on average more often the best alternative is the first. 100 series of Monte Carlo experiments were carried out by 100 in each series. The number of alternatives varied $m = 3, 4, \dots, 50$. We considered the following indicator to compare methods performance: $E_n = \frac{p-m}{p+m+n}$. Here p is the number of experiments with the best the first alternative (the right decision), m indicated the best not the first alternative (the wrong decision). n indicates the number of experiments when WEBIRA can not set the best alternative (it can be nonzero only for WEBIRA).

One Way ANOVA was performed to compare average values of indicator E_n for the fixed m values at significance level 0.05. The results of the experiment show that if the set of attributes consist of 2 independent subsets of evaluaing criteria, the average efficiency of WEBIRA is significantly higher than efficiency of EMDCW and AVRG methods for all considered number of alternatives regardless of what data normalization method is used.

REFERENCES

[1] A. Krylovas, N. Kosareva and E.K. Zavadskas. Statistical analysis of KEMIRA type weights balancing methods. *Romanian Journal of Economic Forecasting*, **19** (3):19–39, 2016.

[2] E.K. Zavadskas and V. Podvezko. Integrated Determination of Objective Criteria Weights in MCDM. *International Journal of Information Technology & Decision Making*, **15** (2):267–283, 2016.

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CONVERGENCE ORDER IN TRAJECTORY ESTIMATION BY PIECEWISE CUBICS AND EXPONENTIAL PARAMETERIZATION

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In this work we estimate the trajectory of the regular curve γ from the sequence of $m+1$ interpolation points $Q_m = \{q_i\}_{i=0}^m$ (with $q_{i+1} \neq q_i$) in arbitrary Euclidean space E^n satisfying $q_i = \gamma(t_i)$. The corresponding interpolation knots $\mathcal{T}_m = \{t_i\}_{i=0}^m$ fulfilling $t_i < t_{i+1}$ are assumed to be unknown (such Q_m is called *reduced data*). In the first step an interpolation scheme $\hat{\gamma} : [0, \hat{T}] \to E^n$ needs to be selected - here *piecewise Lagrange cubics*. Next the missing knots $\mathcal{T}_m$ should be estimated by some $\hat{\mathcal{T}}_m = \{\hat{t}_i\}_{i=0}^m$ (with $\hat{t}_i < \hat{t}_{i+1}$) defined here according to the *exponential parameterization* which depends on a single parameter $\lambda \in [0,1]$ - see e.g. [1; 2]. Having chosen an interpolation scheme $\hat{\gamma} : [0, \hat{T}] \to E^n$ and the knots' estimates $\{\hat{t}_i\}_{i=0}^m \approx \{t_i\}_{i=0}^m$ the analysis of the intrinsic asymptotic order in γ approximation by $\hat{\gamma}$ can be addressed (for dense data Q_m). The main result of this work reads as (supported also by the proof and the numerical experimentation):

THEOREM 1. *Let a regular γ be C^4 with the unknown interpolation knots $\{t_i\}_{i=1}^m$ sampled more-or-less uniformly (see e.g. [1]). If $\hat{\gamma}$ represents a piecewise-cubic Lagrange interpolant based on reduced data Q_m and exponential parameterization with $\lambda \in [0,1]$, then, for some piecewise-cubic-C^∞ $\psi : [0,T] \to [0,\hat{T}]$:*

$$(\hat{\gamma} \circ \psi)(t) = \gamma(t) + O(\delta_m^1) \text{ for } \lambda \in [0,1) \quad \text{and} \quad (\hat{\gamma} \circ \psi)(t) = \gamma(t) + O(\delta_m^4) \text{ for } \lambda = 1. \tag{1}$$

Here $\delta_m = \max_{0 \leq i \leq m-1}\{t_{i+1} - t_i\}$.

REFERENCES

[1] R. Kozera, L. Noakes. Exponential parameterization and ϵ-uniformly sampled reduced data. *Applied Mathematics and Information Sciences*, **10(1)** 33–48, 2016.

[2] R. Kozera, L. Noakes. Piecewise-quadratics and exponential parameterization for reduced data. *Applied Mathematics and Computation*, **221** 1–19, 2013.

[3] R. Kozera. *Curve modeling via interpolation based on multidimensional reduced data.* Studia Informatica, 25(4B(61)). 1–140, 2004

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THE GENERALIZED MARKET MODEL AND THE ECONOMICS CYCLES

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We consider the well-known mathematical model of macroeconomics, which is called the market model or the aggregate demand-aggregate supply model [1-2]. Traditional versions of this model have the form of one ordinary differential equation

$$\dot{p} = D(p) - S(p), \tag{1}$$

where $p = p(t)$ is the prize of the product at the moment t, $D(p)$ is the demand function, $S(p)$ is the supply function. Usually, these function are assumed with the following properties (see, for example,[1]):

1) nonnegative and smooth functions $S(p), D(p)$ are defined for $p \in (0, \infty)$; 2) if $p \in (0, \infty)$, then $D'(p) < 0, S'(p) > 0$; 3) $\lim\limits_{p\to 0} D(p) = D_0$, where $D_0 >> 1$ or $D_0 = \infty, \lim\limits_{p\to 0} S(p) = 0$; 4) $\lim\limits_{p\to\infty} D(p) = 0$, $\lim\limits_{p\to\infty} S(p) = S_\infty$, where $S_\infty \in R_+$.

In this case, equation (1) has one positive steady $p(t) = p_0 > 0$ and all solutions of equation (1) tend to the equilibrium state $p(t) = p_0$. Hence, the equation (1) can't describe economics cycles.

If we introduce [2,3] a delay and consider the following equation

$$\dot{p} = D(p) - S(p_h),\ p_h = p(t-h),\ h > 0, \tag{2}$$

then it is possible to find the stable solutions, which describe cycles in macroeconomics.

Also, it is possible to study the influence of spatial effects and consider the boundary value problem

$$\dot{p} = D(p) - S(p_h) + dp_{xx}, \tag{3}$$

$$p_x(t, 0) = p_x(t, l) = 0,\ x \in [0, l]. \tag{4}$$

Here $d > 0, p = p(t, x), p_h = p(t-h, x), h > 0$. Sometimes this problem admits spatial nonhomogeneous cycles.

REFERENCES

[1] T. Puu. *Economic Dynamics*. Berlin-Heidelberg,Springer-Verlag, New York, 1990.

[2] A.N. Kulikov, D.A. Kulikov. Mathematical model of the market and effect of delay. *Mathematics in Yaroslavl University: A collection of review articles for the 40th anniversary of the faculty*, **2016, 132 – 151** (in Russian).

[3] D.A. Kulikov. About a mathematical model of market. *Journal of Physics: Conference Series*, **788** 012024 (6pp), 2017.

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IMAGINARY PARTS OF NON-TRIVIAL ZEROS IN THE THEORY OF UNIVERSALITY

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In 1975, S.M. Voronin proved that the Riemann zeta-function $\zeta(s)$, $s = \sigma + it$, is universal in the sense that the shifts $\zeta(s + i\tau)$, $\tau \in \mathbb{R}$, approximate a wide class of analytic functions. Let $\gamma_1 \leqslant \gamma_2 \leqslant \cdots$ be positive imaginary parts of non-trivial zeros of $\zeta(s)$. We consider the approximation of analytic functions by shifts $\zeta(s + i\gamma_k)$, $k \in \mathbb{N}$.

In [4], H.L. Montgomery conjectured that

$$\sum_{\substack{0<\gamma,\gamma'\leqslant T\\ |\gamma-\gamma'|<c/\log T}}$$

as $T \to \infty$, is asymptotically equal to $c_1 T \log T$ with some positive c and c_1. We use a weaker form of the above conjecture, we suppose that, for the above sum, the bound $\ll T \log T$ is true. Let, as usual, $D = \left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$, $\mathcal{K}$ be the class of compact subsets of the strip D, and let $H_0(K)$ with $K \in \mathcal{K}$ denote the class of continuous non-vanishing functions on K which are analytic in the interior of K. Then we have

THEOREM 1. *Suppose that the weaker Montgomery hypothesis is true. Let $K \in \mathcal{K}$ and $f(s) \in H_0(K)$. Then, for every $\varepsilon > 0$ and $h > 0$,*

$$\liminf_{N\to\infty} \frac{1}{N} \# \left\{1 \leqslant k \leqslant N : \sup_{s\in K} |\zeta(s + i\gamma_k h) - f(s)| < \varepsilon\right\} > 0.$$

REFERENCES

[1] H.L. Montgomery. The pair correlation of zeros of the zeta function. In: *Analytic Number Theory*, Proc. Sympos. Pure Math., Vol. XXIV, (St. Louis Univ., 1972), H.G. Diamond(Ed.), Amer. Math. Soc., Providence, 1973, 181–193.

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SPATIAL ENTROPY OF MULTI-DIMENSIONAL SYMBOLIC DYNAMICAL SYSTEMS

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The commonly used topological entropy of the multi-dimensional shift space is the rectangular spatial entropy which is the limit of growth rate of admissible local patterns on

nite rectangular sublattices which expands to whole space. In this talk, we study spatial entropy of shift space on general expanding system which is increasing

nite sublattices and expanding to whole space. It can be shown that the spatial entropy is greater than or equal to the rectangular spatial entropy. We prove that the spatial entropy is equal to the rectangular spatial entropy for all shift spaces if and only if the ratio of the size of boundary of the expanding system to the area of the system tends to zero. Therefore, the rectangular spatial entropy is appropriate to measure the complexity of multi-dimensional shift spaces.

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WEAK CONSERVATIVENESS CRITERIONS FOR QGD-SCHEMES FOR 1D GAS DYNAMICS EQUATIONS

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We study explicit symmetric finite-difference schemes for the 1D barotropic gas dynamics system of equations which are based on its special quasi-gas/hydrodynamic (QGD) regularizations [1; 2]. We introduce uniform grids ω_h with nodes $x_k = kh$ and ω_h^* with nodes $x_{k+1/2} = (k+0.5)h$ on $\mathbb{R}$, with $k \in \mathbb{Z}$ and the step $h > 0$, as well as the grid in t with nodes $t_m = m\Delta t$, with $m \geqslant 0$ and the step $\Delta t > 0$. We define the shift, averaging and difference grid operators: $v_{\pm,k} = v_{k\pm1}$, $(sv)_{k-1/2} = (v_k + v_{k+1})/2$, $(\delta v)_{k-1/2} = (v_k - v_{k-1})/h$, $(\delta^* y)_k = (y_{k+1/2} - y_{k-1/2})/h$, $\delta_t v = (\hat{v} - v)/\Delta t$ and $\hat{v}^m = v^{m+1}$. In particular, we consider QGD-difference scheme of the standard type

$$\delta_t \rho + \delta^* j = 0, \quad \delta_t(\rho u) + \delta^*\big(jsu + p(s\rho) - \Pi\big) = 0,$$
$$j = s\rho \cdot su - s\rho \cdot w, \quad s\rho \cdot w = (s\tau)\delta(\rho u) \cdot su + s\rho \cdot \hat{w}, \quad s\rho \cdot \hat{w} = (s\tau)\big[s\rho \cdot su \cdot \delta u + \delta p(\rho)\big],$$
$$\Pi = \mu \delta u + su \cdot s\rho \cdot \hat{w} + (s\tau)p'(s\rho) \cdot \delta(\rho u).$$

Here the sought functions $\rho > 0$ and u (the density and velocity) are defined on ω_h while $j, w, \hat{w}$ and Π are defined on ω_h^*. Also $\tau = \alpha h/\sqrt{p'(\rho)}$ is the regularization parameter (with $\alpha > 0$ and $p'(\rho) > 0$) and $\mu = \alpha_S \tau \rho p'(\rho)$ is the viscosity coefficient (with $\alpha_S \geqslant 0$). After linearization at a constant solution $\rho_* = \text{const}$ and $u_* = 0$, the scheme takes the following form:

$$\hat{\mathbf{y}} = \begin{pmatrix} \alpha\beta & \frac{\beta}{2} \\ \frac{\beta}{2} & \varkappa\alpha\beta \end{pmatrix} \mathbf{y}_- + \begin{pmatrix} 1 - 2\alpha\beta & 0 \\ 0 & 1 - 2\varkappa\alpha\beta \end{pmatrix} \mathbf{y} + \begin{pmatrix} \alpha\beta & -\frac{\beta}{2} \\ -\frac{\beta}{2} & \varkappa\alpha\beta \end{pmatrix} \mathbf{y}_+ \tag{1}$$

with three parameters α, $\beta := c_* \frac{\Delta t}{h}$ with the background sound velocity $c_* = \sqrt{p'(\rho_*)}$ and $\varkappa := \alpha_S + 1 \geqslant 1$, where $\mathbf{y}^m = \begin{pmatrix} \rho^m & u^m \end{pmatrix}^T$ and $\hat{\mathbf{y}}^m = \mathbf{y}^{m+1}$. Let H be the complex Hilbert space of square-summable functions on the grid ω_h.

THEOREM 1. *The linearized scheme* (1) *is weakly conservative, i.e. the bound* $\sup\limits_{m \geqslant 0} \|\mathbf{y}^m\|_H \leqslant \|\mathbf{y}^0\|_H$ *holds for any* $\mathbf{y}^0 \in H$*, if and only if the condition* $\beta \leq \min\big\{2\alpha, 1/(2\varkappa\alpha)\big\}$ *is satisfied.*

Some other schemes are studied as well. This work was supported by the RFBR, project no. 16-01-00048.

REFERENCES

[1] T.G. Elizarova. *Quasi-gas dynamic equations*. Springer, Berlin, 2009.

[2] A.A. Zlotnik. Energy equalities and estimates for barotropic quasi-gasdynamic and quasi-hydrodynamic systems of equations. *Comput. Math. and Math. Phys.*, **50** (2):310–321, 2010.

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ON DISCRETE UNIVERSALITY OF THE RIEMANN ZETA-FUNCTION

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It is well known that the Riemann zeta-function $\zeta(s)$, $s = \sigma + it$, is universal in the Voronin sense, that is, its shifts $\zeta(s + i\tau)$, $\tau \in \mathbb{R}$, approximate a wide class of analytic functions. We will focus on the discrete universality of $\zeta(s)$, when τ takes values from a certain discrete set (e.g., from an arithmetic progression). The simplest discrete result was given by B. Bagchi [1] for the set $\{kh : k \in \mathbb{N}_0\}$, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, where $h > 0$ is a fixed number. His result has been extended by A. Dubickas and A. Laurinčikas in [2] for the sequence $\{k^\alpha h : k \in \mathbb{N}_0\}$ with a fixed α, $0 < \alpha < 1$.

The aim of this talk is a generalization of the mentioned A. Dubickas and A. Laurinčikas' result. More precisely, we will present the discrete universality of $\zeta(s)$ [3] for the class $\mathfrak{X}$ of sequences $\{x_k : k \in \mathbb{N}\} \subset \mathbb{R}$ satisfying the following hypotheses:

1. $\{ax_k\}$ is uniformly distributed modulo 1 for all real $a \neq 0$;
2. $1 \leq x_k \leq k$ for all $k \in \mathbb{N}$;
3. for $1 \leq k, m \leq N$, $k \neq m$, the inequality

$$|x_k - x_m| \geq \frac{1}{y_N}$$

holds with $y_N > 0$ satisfying $y_N x_N \ll N$.

THEOREM 1. *Suppose that the sequence $\{x_k : k \in \mathbb{N}\} \in \mathfrak{X}$. Let $K \subset \left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$ be a compact subset with connected complement, and let $f(s)$ be a continuous non-vanishing function on K which is analytic in the interior of K. Then, for every $h > 0$ and $\epsilon > 0$,*

$$\liminf_{N\to\infty} \frac{1}{N} \# \left\{ 1 \leq k \leq N : \sup_{s\in K} \left|\zeta(s + ix_k h) - f(s)\right| < \epsilon \right\} > 0.$$

REFERENCES

[1] B. Bagchi. *The statistical behaviour and universality properties of the Riemann zeta-function and other allied Dirichlet series*. Ph. D. Thesis. Indian Statistical Institute, Calcutta, 1981.

[2] A. Dubickas and A. Laurinčikas. Distribution modulo 1 and the discrete universality of the Riemann zeta-function. *Abh. Math. Semin. Univ. Hamb.*, **86** (1):79–87, 2016.

[3] R. Macaitienė. On discrete universality of the Riemann zeta-function with respect to uniformly distributed shifts. *Arch. Math.*, (to appear).

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MODELLING THE SPATIOTEMPORAL DYNAMICS OF CHEMOVIROTHERAPY CANCER TREATMENT

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Chemovirotherapy is a combination therapy with chemotherapy and oncolytic viruses. It is gaining more interest and attracting more attention in the clinical setting due to its effective therapy and potential synergistic interactions against cancer. In this paper, we develop and analyse a mathematical model in the form of parabolic non-linear partial differential equations to investigate the spatiotemporal dynamics of tumour cells under chemovirotherapy treatment. The proposed model consists of uninfected and infected tumour cells, a free virus, and a chemotherapeutic drug. The analysis of the model is carried out for both the temporal and spatiotemporal cases. Travelling wave solutions to the spatiotemporal model are used to determine the minimum wave speed of tumour invasion. A sensitivity analysis is performed on the model parameters to establish the key parameters that promote cancer remission during chemovirotherapy treatment. Model analysis of the temporal model suggests that virus burst size and virus infection rate determine the success of the virotherapy treatment, whereas, travelling wave solutions to the spatiotemporal model show that tumour diffusivity and growth rate are critical during chemovirotherapy. Simulation results reveal that chemovirotherapy is more effective and a good alternative to either chemotherapy or virotherapy, which is in agreement with the recent experimental studies.

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ON NUMERICAL SIMULATION OF ELECTROMAGNETIC FIELD EFFECTS IN THE COMBUSTION PROCESS

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The present talk considers a simplified model taking into account the interplay of compressible, laminar, axisymmetric flow and the electrodynamical effects due to Lorentz force's action on the combustion process in a cylindrical pipe.
The flow with axial, radial and azimuthal components u_z, u_r, u_ϕ of velocity is developed. The circulation $v = ru_\phi$ of azimuthal velocity with rotation of tube inlet's part is formed. Similar experiment is considered in [1].
The combustion process with Arrhenius kinetics is modelled by a single step exothermic chemical reaction of fuel and oxidant. Direct electric current with the meridian components of the uniform distribution of current density is fed to 2 axially-symmetric conductor-electrodes(the walls of the pipe and the central part of the pipe). In the ionized gas an electric current creates the azimuthal component B_ϕ of the induced magnetic field, which creates axial F_z and radial F_r components of the electromagnetic force.
We analyze following non-stationary PDEs with 7 unknown functions $\rho, u_r, u_z, v, T, C, B_\phi$:
$\frac{\partial\rho}{\partial t} + u_r\frac{\partial\rho}{\partial r} + u_z\frac{\partial\rho}{\partial z} + \frac{\rho}{r}\frac{\partial(ru_r)}{\partial r} + \rho\frac{\partial u_z}{\partial z} = 0, \quad \frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z} - \frac{v^2}{r^3} = -\frac{\partial p}{\rho\partial r} + F_r,$
$\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z} = -\frac{\partial p}{\rho\partial z} + F_z, \qquad \frac{\partial v}{\partial t} + u_r\frac{\partial v}{\partial r} + u_z\frac{\partial v}{\partial z} = 0,$
$\frac{\partial T}{\partial t} + u_r\frac{\partial T}{\partial r} + u_z\frac{\partial T}{\partial z} = \frac{\lambda}{\rho c_p}\Delta T + \frac{\tilde{B}}{c_p}A\,C\exp(-\frac{E}{RT}), \quad \frac{\partial C}{\partial t} + u_r\frac{\partial C}{\partial r} + u_z\frac{\partial C}{\partial z} = \frac{D}{\rho}\Delta C - A\,C\exp(-\frac{E}{RT}),$
$\frac{\partial(B_\phi)}{\partial t} + \frac{\partial(u_rB_\phi)}{\partial r} + \frac{\partial(u_zB_\phi)}{\partial z} = \nu_m\Delta^*B_\phi,$
where $\Delta q, \Delta^*B_\phi$ are the direct and conjugate Laplace operators $(q = T; C)$, $T, C, \rho, D, \nu_m, \lambda, c_p$ are the temperature, mass fraction of the fuel, density, molecular diffusivity, magnetic viscosity, thermal conductivity and the specific heat,
$\tilde{B}, A, E, R$ are the specific heat release, reaction-rate pre-exponential factor, activation energy and the universal gas constant.For pressure we consider the ideal gas law $p = RT\rho/M'$, where M' is the molar mass for O_2.
For the inviscid flow approximation we use the implicit FDS in time with upwind differences in space. For solving the discrete problem we use the ADI method of Douglas and Rachford [1] .

REFERENCES

[1] I. Barmina, A. Kolmickovs, R. Valdmanis, M.Zake, H.Kalis. Experimental and Numerical Studies of Electric Field Effects on Biomass Thermo-chemical Conversion. *Chemical Engineering Transactions*, **50** 121–126, 2016.

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MODIFIED UNIVERSALITY THEOREMS FOR ZETA-FUNCTIONS

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By the Voronin theorem and its generalizations, it is known that a wide class of analytic functions can be approximated by shifts of zeta-functions. More precisely, the set of shifts approximating a given analytic function has a positive lower density. The results of such a type are called universality theorems. We obtain modified universality theorems which are stated in terms of a positive density with one exception related to the accuracy of approximation. Modified universality theorems for the Riemann zeta-function $\zeta(s)$, $s = \sigma + it$, were obtained in [1] and [4], for the Hurwitz zeta-function $\zeta(s, \alpha)$ in [2], and joint theorems for $\zeta(s)$ and $\zeta(s, \alpha)$ in [3]. As an example, we state a modified joint discrete theorem for $\zeta(s)$ and $\zeta(s, \alpha)$.

Let $D = \left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$, $\mathcal{K}$ be a class of compact subsets of the strip D with connected complements, $H(K)$, $K \in \mathcal{K}$, be the class of continuous functions on K which are analytic in the interior of K, and let $H_0(K)$, $K \in \mathcal{K}$, be the subclass of $H(K)$ of non-vanishing functions on K.

THEOREM 1. *Suppose that the set* $\left\{(\log p : p \in \mathbb{P}), (\log(m + \alpha) : m \in \mathbb{N}_0), \frac{\pi}{h}\right\}$ *is linearly independent over the field of rational numbers. Let* $K_1, K_2 \in \mathcal{K}$ *and* $f_1(s) \in H_0(K_1)$, $f_2(s) \in H(K_2)$. *Then the limit*

$$\lim_{N\to\infty} \frac{1}{N+1} \# \left\{0 \leqslant k \leqslant N : \sup_{s\in K_1} |\zeta(s + ikh) - f_1(s)| < \varepsilon, \right.$$
$$\left. \sup_{s\in K_2} |\zeta(s + ikh, \alpha) - f_2(s)| < \varepsilon \right\} > 0$$

exists for all but at most countably many $\varepsilon > 0$.

REFERENCES

[1] A. Laurinčikas and L. Meška. Improvement of the universality inequality. *Math. Notes*, **96** (5-6):971 – 976, 2014.

[2] A. Laurinčikas and L. Meška. On the modification of the universality of the Hurwitz zeta-functions. *Nonlinear Analysis: Modell. Contr.*, **21** (4):564 – 576, 2016.

[3] A. Laurinčikas and L. Meška. A modification of the Mishou theorem. *Chebysh. Sb.*, **17** (3):125 – 137, 2016.

[4] L. Meška. A modification of the universality inequality. *Šiauliai Math. Semin.*, **9(17)** :71 – 81, 2014.

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ON THE DISCRETE UNIVERSALITY OF THE LERCH ZETA-FUNCTION

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Let $s = \sigma + it$ be a complex variable, $\alpha \in [0,1]$ and $\lambda \in \mathbb{R}$ be fixed parameters. The Lerch zeta-function $L(\lambda, \alpha, s)$ is defined, for $\sigma > 1$, by the series

$$L(\lambda, \alpha, s) = \sum_{m=0}^{\infty} \frac{e^{2\pi i \lambda m}}{(m+\alpha)^s},$$

and is meromorphically continued to the whole complex plane. For $\lambda \in \mathbb{Z}$, the function $L(\lambda, \alpha, s)$ reduces to the Hurwitz zeta - function $\zeta(s, \alpha)$.

The function $L(\lambda, \alpha, s)$, as the majority of classical zeta and L-function is universal for some classes of the parameters λ and α in the sense that its shifts $L(\lambda, \alpha, s + i\tau)$, $\tau \in \mathbb{R}$, approximate a wide class of analytic functions. Let $L(\alpha) = \big\{\log(m+\alpha) : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\big\}$. In [1], an universality theorem for $L(\lambda, \alpha, s)$ with α such that the set $L(\alpha)$ is linearly independent over the field of rational numbers has been obtained. This report is denoted to a discrete version of the mentioned theorem.

Let $D = \big\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\big\}$, $\mathcal{K}$ be the class of compact subsets of D with connected complements, and let $H(K)$ with $K \in \mathcal{K}$ be the class of continuous functions on K which are analytic in the interior of K. Denote by $\#A$ the cardinality of the set A. Then the following assertion is true.

THEOREM 1. *Suppose that $h > 0$ and $\alpha \in (0,1]$ are fixed numbers such that the set $\big\{(\log(m+\alpha) : m \in \mathbb{N}_0), \frac{\pi}{h}\big\}$ is linearly independent over $\mathbb{Q}$. Let $K \in \mathcal{K}$ and $f(s) \in H(K)$. Then, for every $\varepsilon > 0$ and $\lambda \in (0,1]$,*

$$\liminf_{N\to\infty} \frac{1}{N+1} \# \left\{ 0 \leqslant k \leqslant N : \sup_{s\in K} |L(\lambda, \alpha, s + ikh) - f(s)| < \varepsilon \right\} > 0.$$

For the proof of Theorem 1, a probabilistic approach is applied.

REFERENCES

[1] A. Mincevič and A. Vaiginytė. Remarks on the Lerch zeta-function. *Šiauliai Math. Semin.*, **11 (19)** :65–73, 2016.

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GENERALIZATIONS OF UNIVERSALITY THEOREMS FOR PERIODIC HURWITZ ZETA-FUNCTIONS

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Let α, $0 < \alpha \leq 1$, be a fixed parameter, and let $\mathfrak{a} = \{a_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$, $s = \sigma + it$, is defined, for $\sigma > 1$, by the series

$$\zeta(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s},$$

and is meromorphically continued to the whole complex plane. As the majority of zeta-functions, the function $\zeta(s, \alpha; \mathfrak{a})$ is universal in the Voronin sense: its shifts $\zeta(s + i\tau, \alpha; \mathfrak{a})$, $\tau \in \mathbb{R}$, approximate analytic functions from a wide class.

Let $D = \{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$. Denote by $H(D)$ the space of analytic functions on D endowed with the topology of uniform convergence on compacta. In the report, we consider universality of composite functions $F(\zeta(s, \alpha; \mathfrak{a}))$ for some classes of operators $F : H(D) \to H(D)$. We give one example. Let $a_1, \ldots, a_r \in \mathbb{C}$, and

$$H_{a_1,\ldots,a_r}(D) = \{g \in H(D) : (g(s) - a_j)^{-1} \in H(D), j = 1, \ldots, r\}.$$

Moreover, let $\mathcal{K}$ be the class of compact subsets of D with connected complements, and $H(K)$, $K \in \mathcal{K}$, denote the class of continuous functions on K which are analytic in the interior of K. Then the following assertion is true.

THEOREM 1. *Suppose that α is a transcendental, and that $F : H(D) \to H(D)$ is a continuous operator such that $F(H(D)) \supset H_{a_1,\ldots,a_r}(D)$. For $r = 1$, let $K \in \mathcal{K}$, $f(s) \in H(K)$ and $f(s) \neq a_1$ on K. For $r \geq 2$, let $K \subset D$ be an arbitrary compact set, and $f(s) \in H_{a_1,\ldots,a_r}(D)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T} \operatorname{meas} \left\{ \tau \in [0, T] : \sup_{s\in K} \left| F(\zeta(s + i\tau, \alpha; \mathfrak{a})) - f(s) \right| < \varepsilon \right\} > 0.$$

Other universality results can be found in [1].

REFERENCES

[1] A. Laurinčikas and D. Mochov. Generalisations of universality theorems for periodic Hurwitz zeta-functions. *Researches Math. Mech.*, **21** (1):92–99, 2016.

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INFINITE PRANDTL NUMBER TWO-DIMENSIONAL NON-UNIFORMLY INTERNALLY HEAT DRIVEN CONVECTION ON $\mathbb{T}^2$

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Infinite Prandtl number convection is modeled by a non-local heat equation obtained from the Navier-Stokes-Boussineq equations without an inertial term

$$T_t + \mathcal{R}\left[\nabla^{\perp}(\Delta^{-2}T_x)\right]\cdot\nabla T = \Delta T + f(x,z) \tag{1}$$

where $T(x,z)$ is the dimensionless temperature, f is a periodic and mean zero distributed internal heating source and sink, and $\mathcal{R}$ is the Rayleigh number which is a dimensionless measure of the heating strength. Previous numerical studies have examined periodic configurations to allow use of the Fourier transform to easily solve this equation[1; 2; 3; 4]. Here, a mean zero forcing is used so that relevant quantities remain bounded for long times. A proxy Nu_h for measuring effective heat transport is

$$1 \leq Nu_h = \frac{\|\nabla^{-1}f\|_2}{\left(\overline{\|\nabla T\|_2^2}\right)^{\frac{1}{2}}} \lesssim C + \mathcal{R}^{\frac{1}{2}}\frac{\|\nabla f\|_{\infty}^{\frac{1}{2}}\|\nabla^{-1}f\|_2}{4\pi^2\|f\|_2^{\frac{1}{2}}} \tag{2}$$

where $\bar{\cdot}$ is a time average. Simulations with various choices of f give results for which Nu_h scales with $\mathcal{R}^{\alpha}$ for $0 \leq \alpha \leq 0.5$.

REFERENCES

[1] V. Borue and S. Orszag. Turbulent convection driven by a constant temperature gradient. *J. Scientific Computing*, **12** 305, 1997.

[2] E. Calzavarini, C. Doering, J. Gibbon, D. Lohse, A. Tanabe, and F. Toschi. Exponentially growing solutions in homogeneous Rayleigh-Bénard convection. *Phys. Rev. E*, **035301** 2006.

[3] E. Calzavarini, D. Lohse, F. Toschi, and R. Tripiccione. Rayleigh and Prandtl number scaling in the bulk of Rayleigh-Bénard turbulence. *Phys. Fluids*, **17** 055107, 2005.

[4] D. Lohse and F. Toschi. Ultimate state of thermal convection. *Phys. Rev. Lett.*, **90** 034502, 2003.

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THE LEONTIEF'S BLACK BOX REVERSE-ENGINEERED

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The present paper attempts to reconstruct what goes inside the black box of the fixed proportions production function. The approach is based on an idea that the conversion of inputs to outputs in an economy occurs in much the same manner as does the conversion of substrates into different molecules in enzyme-catalyzed biochemical reactions. Indeed, in a living cell, a substrate molecule binds to an enzyme to form a substrate-enzyme complex. The complex then breaks up into a product and the original enzyme, which can then catalyze a new reaction. In any link of a production chain, tools and equipment play role of enzymes. Assuming that when two or more factors of production are involved in an interaction step, the rate of process is proportional to the product of their amounts, the differential equations corresponding to a two-resource, one-product technology would be

$$\begin{aligned} \mathrm{d}x_1/\mathrm{d}t &= r_1 - a_1 u x_1 x_2 - q_1 x_1, \\ \mathrm{d}x_2/\mathrm{d}t &= r_2 - a_2 u x_1 x_2 - q_2 x_2, \\ \mathrm{d}u/\mathrm{d}t &= bv - a u x_1 x_2, \\ \mathrm{d}v/\mathrm{d}t &= a u x_1 x_2 - bv, \\ \mathrm{d}p/\mathrm{d}t &= hv. \end{aligned}$$

Here t means time, x_i represents the total amount of ith resource, u and v are the respective numbers of pieces of unloaded and loaded equipment, p is the total amount of the product, the constant r_i defines the feed rate of ith resource, a_i is the capture rate of a unit of ith resource by a piece of equipment per unit of the complementary resource j, q_i is the loss rate of ith resource, a is the rate of loading of each individual piece of equipment per pair of units of the inputs, b is the rate of discharge of a piece of equipment, and h is the rate of production of a unit of the commodity. All parameters in the model are nonnegative. Besides, the total of busy and idle pieces of equipment is a constant: $u + v = u_0$.

It is reasonable that the loading and unloading of equipment is much faster than other rates. The presence of two time scales in the model enables to reduce it to a singularly perturbed system with small parameter $\varepsilon = a_1 u_0 (r_2/(r_1 ab))^{1/2} \ll 1$ at the time derivative of the normalized equipment variable.

Assuming the weakness of the resource outfluxes, $q_1 q_2 \ll a_1 a_2 b u_0^2/a$, we can carry out the complete stability analysis for all three possible steady-state modes of operation of the model. The results show that the output would be proportional to $\min\{ar_1/(a_1 u_0 b), ar_2/(a_2 u_0 b), 1\}$. In other words, the output is controlled not by the total influx of resources available, but by the scarcest resource (limiting factor). Saturation naturally results from the limited number of equipment pieces and the limited rate at which they can operate. And this is exactly what the Leontief production function is all about.

The results can be extended to multiple resources and production chains.

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EMBEDDED WAVEGUIDES RESPONSE TO EXTERNAL EXCITATION

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The paper deals with a simulation of an elastic wave in homogenous isotropic embedded waveguide with generic cross-section using semi analytical finite element (SAFE) formulation. The wave is considered a travelling displacement field in the waveguide as a result of forcing excitation and is expressed via modes superposition [1]:

$$\mathbf{u}(y,x,z,t) = \mathbf{N}(x,y)\sum_{m=1}^{N} -\frac{\mathbf{W}_m\mathbf{P}\mathbf{v}_m}{\mathbf{W}_m\mathbf{B}\mathbf{V}_m}e^{i(k_m(z-z_0)-\omega t)} \tag{1}$$

The solutions for modes are obtained solving SAFE governing eigen problem. Attenuation of the medium in SAFE framework differently from prior researchers is simulated via Rayleigh damping. It is shown, that severe damping is not properly supported by the SAFE formulation. Presented SAFE framework includes assumptions

$$k \approx k^* \tag{2}$$

that allow to simplify the governing equation, which is generally accepted by many researchers. However, evidence is found, that the governing equation likely doesnt properly support moderate and heavy damping. Further research is required to explore possibilities of extending the framework to fully accept general linear damping.

REFERENCES

[1] T. Hayashi, W.J. Song and J.L. Rose. Guided wave dispersion curves for a bar with an arbitrary cross-section, a rod and rail example. *Ultrasonics*, **41** 3, 2003.

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PERIODIC SPLINE INTERPOLATION AND HISTOPOLATION

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For a given grid Δ of knots $a = x_0 < x_1 < ... < x_n = b$ we consider the spline space $X_m(\Delta) = \{S \in C^{m-1}[a,b] \big| S : [x_{i-1}, x_i] \to \mathbb{R}$ is a polynomial at most degree m for each $i = 1, ..., n\}$. Let $X_{p,m}(\Delta) = \{S \in X_m(\Delta) \big| S^{(j)}(a) = S^{(j)}(b), i = 0, ..., m-1\}$ be the space of periodic splines. Then $\dim X_m(\Delta) = n + m$ and $\dim X_{p,m}(\Delta) = n$. The periodic histopolation problem consists of finding $S \in X_{p,m}(\Delta)$ such that with $h_i = x_i - x_{i-1}$

$$\int_{x_{i-1}}^{x_i} S(x)dx = h_i z_i, \quad i = 1, ..., n,$$

for given numbers z_i. This problem is equivalent to the interpolation problem of finding $\bar{S} \in X_{m+1}(\Delta)$ such that $\bar{S}^{(j)}(a) = \bar{S}^{(j)}(b)$, $j = 1, ..., m$, $\bar{S}(x_i) = y_i$, $i = 0, ..., n$, where $y_i = y_{i-1} + h_i z_i$, $i = 1, ..., n$, $y_0 \in \mathbb{R}$ is chosen arbitrarily. Then we get S as the derivative $\bar{S}'$ of $\bar{S}$. Generally, it may be that $\bar{S}$ does not belong to $X_{p,m+1}(\Delta)$.

There are classical works by Meinardus, Merz and ter Morsche from 1974 about the existence and uniqueness of solution for the periodic interpolation problem in the case of equidistant knots. Some works treat the problem of interpolation or histopolation for general grid and for particular spline degree, e. g., Kobza and Ženčák 1997, at histopolation for $m = 4$ but with definite existence and uniqueness results only for uniform grid. We will discuss the existence and uniqueness problem in general case for arbitrary n and m.

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MODELING GASIFICATION/COMBUSTION PROCESS FOR THERMO-CHEMICAL CONVERSION OF BIOMASS MIXTURES[1]

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It is very difficult to predict results of experimental studies that have been obtained for a specific fuel mixture to the biomass mixtures with different elemental and chemical composition of components. Therefore a more fundamental research with mathematical modeling and numerical analysis of the processes is required to understand the main mechanisms determining the development of the gasification/combustion characteristics of biomass mixtures. [1]

In this study we focus on the combustion characteristics of the biogas, where the main combustible volatiles are CO and H_2 . The experimental section of this study has demonstrated that the rate of formation of the volatile gases in the gasifier depends on the stage of gasification process as well as the composition of the biomass. The production of heat and emissions in the burner section is controlled by the composition and mass flow rate of the volatile gases. We develop a model in the environment of ANSYS Fluent CFD package, two exothermic reactions for combustion of H_2 and CO are used.

The simulations give an insight in the combustion process, giving the temperature and velocity distributions in the flame. Further studies are required to couple the gasification stage to the combustion stage.

REFERENCES

[1] M. Abricka, I. Barmina, R. Valdmanis, M. Zake, H. Kalis. Experimental and numerical studies on integrated gasification and combustion of biomass. *Chemical Engineering Transactions*, **50** 127-132, 2016.

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GREEN'S MATRICES FOR FIRST ORDER DIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITIONS

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We investigate the linear system of first order ordinary differential equations with nonlocal conditions

$$\frac{d\,u_i}{dx} = \sum_{j=1}^{n} a_{ij}(x)u_j + f_i(x), \quad i = \overline{1,n}, \quad x \in [0,1],$$
$$\sum_{i=1}^{n} \langle L_{ki}, u_i \rangle = g_k, \quad k = \overline{1,n},$$

where $u_i \in C^1[0,1]$, $f_i \in C[0,1]$, $g_k \in \mathbb{R}$, $L_{ki} \in C^*[0,1]$, $a_{ij} \in C[0,1]$, $i,j,k = \overline{1,n}$.

Green's matrices, their explicit representations and properties are considered as well. We present the relation between the Green's matrix for the system with nonlocal conditions and the Green's function for the differential equation with nonlocal conditions

$$u^{(n)} + a_{n-1}(x)u^{(n-1)} + \ldots + a_1(x)u' + a_0(x)u = f(x), \ x \in [0,1],$$
$$\langle L_k, u \rangle := \sum_{i=1}^{n} \langle L_{ki}, u^{(i-1)} \rangle = g_k, \quad k = \overline{1,n},$$

where $a_j, f \in C[0,1]$, $j = \overline{0,n-1}$, and $L_{ki} \in C^*[0,1]$, $g_k \in \mathbb{R}$, $i,k = \overline{1,n}$.

Several examples are also given.

REFERENCES

[1] R.C. Brown. Generalized Green's functions and generalized inverses for linear differential systems with Stieltjes boundary conditions. *J. Differential Equations*, **16** (2):335–351, 1974.

[2] R.N. Bryan. A linear differential system with general linear boundary conditions. *J. Differential Equations*, **5** (1):38–48, 1969.

[3] W.R. Jones. Differential systems with integral boundary conditions. *J. Differential Equations*, **3** (2):191–202, 1967.

[4] G. Paukštaitė, A. Štikonas. Green's Matrices for First Order Differential Equations with Nonlocal Conditions. *Math. Model. Anal.*, **22** (2):213–227, 2017.

[5] S. Roman. Linear differential equation with additional conditions and formulae for Green's function. *Math. Model. Anal.*, **16** (3):401–417, 2011.

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INVESTIGATION OF FUČÍK TYPE SPECTRUM FOR PROBLEM WITH NONLOCAL INTEGRAL BOUNDARY CONDITION

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Let us consider the Fučík equation

$$-x'' = \mu x^+ - \lambda x^-, \qquad t \in [0,1], \tag{1}$$

with classical boundary conditon in the left side and nonlocal integral boundary condition in the right side of the interval

$$x(0) = 0, \tag{2}$$

$$x(1) = \gamma \int_0^\xi x(s)\mathrm{d}s \tag{3}$$

with the parameters $\mu, \lambda, \gamma \in \mathbb{R}$ and $\xi \in [0,1]$. The Fučík type spectrum for the problem (1)-(2) and nonlocal integral condition with $\xi = 1$ was investigated in [1]. The spectrum of the Sturm-Liouville problem with boundary conditions (2)-(3) was analysed in [2].

In this study we investigate the separate cases of the problem (1) - (3) for the particular values of nonlocality parameters ξ and γ. There are presented analytical expressions and graphs of Fučík type spectrum in all investigated cases.

REFERENCES

[1] N. Sergejeva. On some Problems with Nonlocal Integral Condition. *Mathematical Modelling and Analysis*, **15** (1):113–126, 2010.

[2] S. Pečiulytė. Investigation and Numerical Analysis of Boundary Problems with Various Types Nonlocal Conditions *PhD dissertation*. Vilnius Gediminas Technical University, 2007.

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NUMERICAL STUDY OF SHIELDING CAPABILITIES OF A CURVILINEAR MAGNETIC-FLUID LAYER IN EXTERNAL MAGNETIC FIELD

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The problem of the propagation of magnetic fields through magnetizable material media, in particular, through curvilinear shields and shells of various shapes is of great theoretical and practical interest. At present, active numerical studies of the shielding of magnetic fields by solid thin-walled shells are carried out. However, the shielding capabilities of magnetic fluids with nonlinear magnetic properties, have not been studied up to now.

The problem on the shielding of external uniform magnetic field by means of a cylindrical thick-walled magnetic-fluid layer is considered in this work. The coupled problem consists of three magnetostatics subproblems for the magnetic potential – the linear subproblems in inner and outer nonmagnetic domains, and the nonlinear subproblem in the magnetic fluid layer. The subproblems are related to each other by the transmission conditions on the interfaces between magnetic and nonmagnetic media. The linear subproblems are described by the Laplace equation. Therefore, the boundary element method [1] is used to solve the linear subproblems, which is the most effective in this case. The boundary element method does not need a mesh in the inner and outer nonmagnetic domains and exactly satisfies the condition at the infinity. The nonlinear subproblem for the magnetic potential in the magnetic-fluid layer is formulated in polar coordinates and approximated on a uniform mesh by the finite-difference approach with the second order.

We organized the computational process in the form of an iterative algorithm. Three mesh problems are solved with respect to the magnetic potential independently of each other at every iteration. Two linear systems of the boundary-element equations are solved by the Gaussian elimination method and the nonlinear difference problem for the potential in the magnetic-fluid layer – by the iterative Seidel-type method.

The efficiency of the magnetofluid shield is determined by the efficiency coefficient K_{ef} which shows how many times the external magnetic field reduces in the inner region. The influence of the initial magnetic fluid susceptibility and the thickness of the magnetofluid layer on the coefficient K_{ef} over a wide range of magnetic field strength is investigated. It is established that the magnetofluid shields can be effective in weak and moderate magnetic fields with the intensity of $< 10^5$ A/m.

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REFERENCES

[1] C.A. Brebbia, J.C.F. Telles, L.C. Wrobel. *Boundary Element Techniques.* Springer-Verlag, Berlin, 1984.

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SIMULATION-BASED VALIDATION OF MATHEMATICAL MODELS FOR MAGNETIC-PARTICLE DIFFUSION IN FERROFLUIDS

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Ferrofluids are stable colloidal suspensions of magnetic nanoparticles in a non-magnetic liquid carrier. The particle rearrangement in magnetic fields is caused by the field gradients and interparticle interactions. Mathematical and numerical modeling of the diffusion process for magnetic particles in ferrofluids is of current interest in ferrohydrodynamics.

A theoretical model of the particle diffusion in a weakly-concentrated ferrofluid assumes no interaction between magnetic particles if a magnetic field is applied. This assumption allows to resolve explicitly an analytical solution of the mass-transfer equation for a ferromagnetic particle concentration in a static limit [1]. Another theoretical model for the diffusion process of particles in a moderately-concentrated ferrofluid is recently developed in [2]. This model takes into account the magnetic dipole-dipole and steric interparticle interactions in a magnetic field. In this case, the mass-transfer equation in a static limit can be reformulated as a nonlinear algebraic equation for the concentration and the field intensity.

The goal is to formulate the mathematical model and to analyze the numerical aspects of the modelling for the problem on equilibrium free-surface shapes of ferrofluids, taking the diffusion process into account. Mathematical model consists of a set of three coupled equations: the Maxwell's equations for the magnetic field, the mass-transfer equation for the magnetic particles in the ferrofluid and the magnetically augmented Young-Laplace equation for the force balance on the free-surface between the ferrofluid and a surrounding non-magnetic medium. Due to different assumptions on the ferropaticle behaviour, three variants of the mathematical model are studied. Model 1 assumes a uniform particle distribution in the ferrofluid, model 2 allows only interaction between particles and the field and model 3 takes additionally into account the interparticle interactions. Numerical tests for models 1–3 are performed minding the problem of ferrofluid-layer instability in the applied uniform magnetic field (the Rosensweig instability) [3; 4].

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REFERENCES

[1] V. Polevikov and L. Tobiska. On the solution of the steady-state diffusion problem for ferromagnetic particles in a magnetic fluid. *Mathematical Modelling and Analysis*, **13** (2):233–240, 2008.

[2] A.F. Pshenichnikov, E.A. Elfimova and A.O. Ivanov. Magnetophoresis, sedimentation, and diffusion of particles in concentrated magnetic fluids. *Journal of Chemical Physics*, **134** 184508, 2011.

[3] O. Lavrova, V. Polevikov and L. Tobiska. Numerical study of the Rosensweig instability in a magnetic fluid subject to diffusion of magnetic particles. *Mathematical Modelling and Analysis*, **15** (2):223–233, 2010.

[4] O. Lavrova, V. Polevikov and L. Tobiska. Modelling and simulation of magnetic particle diffusion in a ferrofluid layer. *Magnetohydrodynamics*, **52** (4):417–430, 2016.

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NUMERIC-SYMBOLIC METHODS FOR SEARCHING RELATIVE EQUILIBRIA IN THE RESTRICTED PROBLEM OF FOUR BODIES

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The restricted many-body problem is a well-known model of Celestial Mechanics (see, for example [1]). This abstract deals with a special case of the restricted four-body problem, where three bodies P_0, P_1, and P_2 of masses m_0, m_1, and m_2, respectively, move uniformly in circular orbits around their common center of mass and their configuration is always collinear. The fourth body P_3 having negligible mass does not influence the motion of the three primaries and moves in their gravitational field. Note that another restricted four-body problem formulated on the basis of the Lagrange triangular configurations was completely investigated in [2; 3].

In the rotating coordinate system the simplest solution of the equations of motion determines equilibrium positions of the body P_3, while the most massive body P_0 is located at the origin, and the x coordinates of the bodies P_1, P_2 are equal to 1 and a, respectively. Searching these equilibrium positions for any values of the two system parameters $\mu_1 = m_1/m_0$, $\mu_2 = m_2/m_0$ includes two steps. First, we have to solve the equation

$$1+\mu_1+\mu_2\left(\frac{a}{|a|^3}+\frac{1-a}{|1-a|^3}\right)=\frac{\mu_1}{a}\left(1+\frac{a-1}{|a-1|^3}\right)+\frac{1+\mu_2}{|a|^3}=\kappa\ , \tag{1}$$

and find equilibrium coordinate a of the body P_2. Second, we solve a system of two equations

$$\begin{gathered}\kappa x-\mu_1-\mu_2\frac{|a|}{a^3}-\frac{x}{(x^2+y^2)^{3/2}}-\frac{\mu_1(x-1)}{((x-1)^2+y^2)^{3/2}}-\frac{\mu_2(x-a)}{((x-a)^2+y^2)^{3/2}}=0\ ,\\ y\left(\kappa-\frac{1}{(x^2+y^2)^{3/2}}-\frac{\mu_1}{((x-1)^2+y^2)^{3/2}}-\frac{\mu_2}{((x-a)^2+y^2)^{3/2}}\right)=0\ .\end{gathered} \tag{2}$$

Note that solutions of equations (1), (2) cannot be found in analytical form. To separate and find different solutions we have to combine symbolic and numerical calculations. As a result, we have found 18 equilibrium configurations of the system and investigated their behavior for different values of parameters μ_1, μ_2. Computational problems arising in this study are discussed in detail.

REFERENCES

[1] V. Szebehely. *Theory of Orbits. The Restricted Problem of Three Bodies*. Academic Press, New York, 1967.

[2] D.A. Budzko, A.N. Prokopenya. On the stability of equilibrium positions in the circular restricted four-body problem. In: *Proc. of the 13th Intern. Workshop CASC-2011, Kassel, Germany, 2011*, Computer Algebra in Scientific Computing, V.P. Gerdt et al. (Eds.), LNCS6885, Springer-Verlag, Berlin Heidelberg, 2011, 88 – 100.

[3] D.A. Budzko, A.N. Prokopenya. Symbolic-numerical methods for searching equilibrium states in a restricted four-body problem. *Programming and Computer Software*, **39** (2):74–80, 2013.

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FAST SOLVERS FOR WEAKLY SINGULAR INTEGRAL EQUATIONS OF THE SECOND KIND

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We discuss a question about construction of fast solvers for weakly singular Fredholm integral equations of the second kind

$$u(x) = \int_0^1 (a(x,y) \mid x-y \mid^{-\nu} + b(x,y))u(y)dy + f(x), \;\; 0 \le x \le 1, \tag{1}$$

where $0 < \nu < 1$, $a, b \in C^{2m}([0,1] \times [0,1])$, $f \in C^{m,\nu}(0,1)$, i.e. $f \in C[0,1] \cap C^m(0,1)$ and

$$\| f \|_{C^{m,\nu}} := \max_{0 \le x \le 1} \mid f(x) \mid + \sum_{k=1}^{m} \sup_{0<x<1} (x(1-x))^{k-1+\nu} \mid f^{(k)}(x) \mid < \infty.$$

We also assume that the corresponding homogeneous integral equation has in $C[0,1]$ only the trivial solution $u = 0$. Then equation (1) has a solution $u \in C^{m,\nu}(0,1)$ which is unique in $C[0,1]$.

By a *fast* $(C, C^{m,\nu})$ *solver* of equation (1) we mean a solver which produces approximate solutions u_n, $n \in \mathbb{N}$, such that

- given the values of a, b and f at not more than $n_\star$ points depending on the solver (with $n_\star \to \infty$ as $n \to \infty$), the parameters of u_n can be determined at the cost of $\gamma_m n_\star$ arithmetical operations and an accuracy

$$\| u - u_n \|_\infty := \sup_{0 \le x \le 1} \mid u(x) - u_n(x) \mid \le c_m n_\star^{-m} \| f \|_{C^{m,\nu}}$$

  is achieved where u is the solution of (1);

- having the parameters of u_n in hand, the value of u_n at any point $x \in [0,1]$ is available at the cost of γ'_m operations;

Here the constants c_m, γ_m, γ'_m are independent of f and n.

We reduce (1) to a periodic problem and use ideas of [1].

REFERENCES

[1] A. Pedas and G. Vainikko. What is the complexity of periodic weakly singular integral equations?. *BIT Num. Math*, **48** 315-335, 2008.

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FINITE-DIFFERENCE SCHEME WITH LOCAL GRID REFINEMENT

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This paper presents a problem-oriented approach to the numerical simulation of elastic wave propagation in media with small-scale heterogeneities. This property requires a special treatment increasing the computational complexity of an algorithm. At the same time, such formations typically fill relatively small volume of the model, thus the local use of such refinement can speed-up the simulation essentially. In the paper we discuss both mathematical and numerical aspects of the hybrid algorithm paying most attention to its parallel implementation. At the same time essential efforts are spent to couple different equations and, hence, different finite-difference stencils to describe properly the different nature of elastic wave propagation in different areas. The main issue in the coupling is to suppress numerical artifacts down to the acceptable level.

The main purpose of the algorithm is to study scattering and diffraction of elastic waves by clusters of small scale heterogeneities. In such a situation, a detailed description of each fracture is impossible. Hence we assume that the reservoir model is given on a sufficiently fine grid, which possesses grid steps of the size of several centimeters. A typical seismic wave has a wavelength of about several dozen meters, with the grid steps of the background model being about several meters. Thus, a local mesh refinement is used to perform the full waveform simulation of long wave propagation through the medium with a fine structure. As a result, the problem of simulation of seismic wave propagation in models containing small-scale structures becomes a mathematical problem of the local time-space mesh refinement.

The use of the local space-time grid stepping makes difficult to ensure a uniform work load for processors in the domain decomposition. The parallel computation is implemented using two groups of processors. The 3D heterogeneous environment (a coarse grid) is placed on one group, while the fine mesh describing the reservoir is distributed in the other group. There is a need for interactions between processors within each group and between the groups as well. The data exchange within a group is done via faces of the adjacent subdomains by non-blocking iSend/iReceive MPI procedures. The interaction between the groups is designed for coupling a coarse and a fine grids.

From coarse to fine. The processors aligned to a coarse grid are grouped along each of the faces contacting the fine grid. At each of the faces, Master Processor (MP) gathers the computed current values of stresses/displacements, applies the FFT and sends a part of the spectrum to the relevant MP on a fine grid. All the subsequent data processing, including interpolation and inverse FFT, are performed by the relevant MP in the fine grid group. Subsequently, this MP sends interpolated data to each processor in its subgroup.

From fine to coarse. Processors from the second group compute the solution on the fine grid. For each face of the fine grid block a MP is identified. This MP collects data from a relevant face, performs the FFT, and sends a part of the spectrum to the corresponding MP of the first group (a coarse grid). Formally, the FFT can be excluded and the data to be exchanged can be obtained as a projection of the fine grid solution onto the coarse grid, however the use of truncated spectra decreases the amount of data to be exchanged and ensures stability as it suppress high frequencies.

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INVESTIGATIONS OF THE ASYMPTOTIC BEHAVIOUR OF PERIODIC HURWITZ ZETA-FUNCTIONS

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Let $s = \sigma + it$ be complex variable, $\mathfrak{a} = \{a_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers, and α, $0 < \alpha \leq 1$, be a fixed parameter. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$ is defined, for $\sigma > 1$, by the series

$$\zeta(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s},$$

and is meromorphically continued to the whole complex plane.

We characterize the asymptotic behaviour of the function $\zeta(s, \alpha; \mathfrak{a})$ by limit theorem on the weak convergence of probability measures on the complex plane. The properties of the function $\zeta(s, \alpha; \mathfrak{a})$ depend on the arithmetic of the parameter α. We consider separately the cases of transcendental, rational and algebraic irrational α. In all limit theorems, the limit measures are explicitly given. Moreover, two types, continuous and discrete, of limit theorems are considered. The main results are published in [1], [2], [3], [4], [5] and [6].

REFERENCES

[1] D. Genienė and A. Rimkevičienė. A joint limit theorem for periodic Hurwitz zeta-functions with algebraic irrational parameters. *Math. Modell. Analysis*, **18** (1):149 – 159, 2013.

[2] G. Misevičius and A. Rimkevičienė. Joint limit theorems for periodic Hurwitz zeta-function. II. *Annales Univ. Sci. Budapest., Sect. Comp.*, **41** :173 – 185, 2013.

[3] A. Rimkevičienė. Limit theorems for the periodic Hurwitz zeta-function. *Šiauliai Math. Semin.*, **5(13)** :55 – 69, 2010.

[4] A. Rimkevičienė. Joint limit theorems for the periodic Hurwitz zeta-functions. *Šiauliai Math. Semin.*, **6(14)** :53 – 68, 2011.

[5] A. Rimkevičienė. A discrete limit theorem for the periodic Hurwitz zeta-function. In: *Proc. of the Lithuanian Mathematical Society*, Ser. A, Vol. **56**, 2015, 90 – 94.

[6] A. Rimkevičienė. A discrete limit theorem for the periodic Hurwitz zeta-function II. In: *Proc. of the Lithuanian Mathematical Society*, Ser. A, Vol. **57**, 2016, 71 – 74.

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ON SYSTEMS ARISING IN GENE REGULATION THEORY

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Two ordinary differential equation models of the two-component networks [1] are considered. The comparison is made of the model appearing in the work [3]

$$\begin{cases} x_1' = k_1 h(x_2, \Theta_2, \mu_2) - \gamma_1 x_1, \\ x_2' = k_2 h(x_1, \Theta_1, \mu_1) - \gamma_2 x_2, \end{cases} \tag{1}$$

where $h(x, \Theta, \mu) = \dfrac{\Theta^\mu}{x^\mu + \Theta^\mu}$, and the model that has been studied in [2]

$$\begin{cases} x_1' = \dfrac{1}{1 + e^{-\mu(-x_2 - \theta)}} - x_1, \\ x_2' = \dfrac{1}{1 + e^{-\mu(-x_1 - \theta)}} - x_2. \end{cases} \tag{2}$$

We study configurations of critical points and their dependence on parameters. These combinations of critical points are interpreted as attractors. Illustrations and examples are considered also.

REFERENCES

[1] F.M. Alakwaa. Modeling of Gene Regulatory Networks: A Literature Review. *Journal of Computational Systems Biology*, **1** (1), 67-103, 2002.

[2] S. Atslega, D. Finaskins, F. Sadyrbaev. On a Planar Dynamical System Arising in the Network Control Theory. *Mathematical Modelling and Analysis*, **21** (3):385 - 398, 2016.

[3] K.W. Kohn. Molecular interaction maps as information organizers and simulation guides. *Chaos*, **11** (1):8497, 2001.

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PERIODIC POLYNOMIAL SPLINE HISTOPOLATION

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For a given grid Δ of knots, $a = x_0 < x_1 < \ldots < x_n = b$ we consider a spline space $X_m(\Delta) = \{S|S|_{[x_{i-1},x_i]} \in P_m$ (set of all polynomials of degree at most m), $S \in C^{m-1}[a,b]\}$ with the $\dim X_m = n + m$. The space $X_{p,m}(\Delta)$ of peroidic splines is

$$X_{p,m}(\Delta) = \{S \in X_m | S^{(j)}(a) = S^{(j)}(b), \quad j = 0, 1, \ldots, m-1\}.$$

and $\dim X_{p,m}(\Delta) = n$. In histopolation problem we have to find $S \in X_{p,m}(\Delta)$ such that

$$\frac{1}{h_i} \int_{x_{i-1}}^{x_i} S(x)dx = z_i, \quad i = 1, \ldots, n,$$

where $h_i = x_i - x_{i-1}$ and z_i are given real numbers. We will study the problem of existence and uniqueness of solution for such problems.

For m even the solution exists and is unique. This could be proved by using ideas from the variational theory of splines. The solution also exists and is unique when both m and n are odd. The proof of this assertion is based on the estimation of sign change zeros of periodic splines from $X_{p,m}(\Delta)$. The case m odd and n is even seems to be the most complicated and we can prove here nonuniqueness for some particular cases on m and n. These results were known in case of uniform grids.

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DEVELOPMENT AND STUDY OF RELIABLE DIFFERENCE SCHEMES FOR THE CAUCHY PROBLEM FOR A SYSTEM OF TWO SINGULARLY PERTURBED ORDINARY DIFFERENTIAL EQUATIONS

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Effective numerical methods for solving the Cauchy problem in the case of a system of regular ordinary differential equations are well developed (see, e.g., [1] – [5] and the references therein). However, in the case of singularly perturbed ordinary differential equations, i.e., equations with a perturbation parameter ε multiplying the derivatives where the parameter ε takes arbitrary values in the subinterval $(0, 1]$, these methods are unapplicable because the boundary layer appearing in the solution.

In the talk, we consider the development of a reliable difference scheme, i.e., a scheme convergent ε-uniformly, for the Cauchy problem for a system of two singularly perturbed ordinary differential equations. The reliable difference scheme is constructed on a special grid, which is piecewise-uniform in x, condensed in a neighborhood of the boundary layer. It is established that such a scheme converges ε-uniformly in the maximum norm at the rate of $\mathcal{O}(N^{-1}lnN)$ as $N \to \infty$, where N is the number of grid intervals in x.

Justification of ε-uniform convergence of the schemes carried out with the use of derived a priori estimates for the regular and singular components of the problem solution.

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REFERENCES

[1] N. Bakhvalov. *Numerical Methods*. Mir Publishers, Moscow, 1976.

[2] N. Kalitkin. *Numerical Methods*. Nauka, Moscow, 1978. (in Russian)

[3] N. Kalitkin, P. Koriakin. *Numerical methods. Methods of mathematical physics*. Publishing Center "Academy", Moscow, 2013. (in Russian)

[4] G. Marchuk. *Methods of Numerical Mathematics*. Springer–Verlag, New York–Berlin, 1982.

[5] A. Samarskii. *Theory of Difference Schemes*. Marcel Dekker, New York, 2001.

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NUMERICAL STUDY OF DIFFERENCE SCHEMES FOR A MODEL CAUCHY PROBLEM FOR A SINGULARLY PERTURBED ORDINARY DIFFERENTIAL EQUATION

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In the talk, a model Cauchy problem is considered for a singularly perturbed ordinary differential equation with constant coefficients

$$Lu(x) \equiv \varepsilon\, a\, \frac{d}{dx}u(x) + b\, u(x) = f(x),\ x \in D, \quad u(x) = \varphi,\ x \in \Gamma; \tag{1}$$

$$a,\, b > m, \quad x \in \overline{D}; \quad \overline{D} = D \cup \Gamma, \quad D = (0 < x \le d], \quad \Gamma = \{x = 0\}.$$

Here ε is a perturbation parameter, $\varepsilon \in (0, 1]$. For small values of the parameter ε, a boundary layer of width $\mathcal{O}(\varepsilon)$ appears in the solution of this problem. We are interested in the behavior of grid solutions of this problem, namely, the behavior of its errors depending on the number of grid nodes and the parameter ε.

For a given model Cauchy problem, a standard difference scheme on a uniform grid and a special difference scheme on a piecewise-uniform grid are constructed and investigated. Numerical experiments have shown that the solution of the standard difference scheme on a uniform grid does not converge ε-uniformly in the maximum norm while the solution of the special difference scheme on a piecewise-uniform grid converges ε-uniformly in the maximum norm at the rate of $\mathcal{O}(N^{-1}lnN)$ as $N \to \infty$, where N is the number of grid intervals in x. Results of the numerical experiments are consistent with the theoretical results.

The results of this study are basic for the development of ε-uniformly convergent difference schemes for a system of two singularly perturbed ordinary differential equations.

This research was partially supported by the Russian Foundation for Basic Research under grant No. 16-01-00727.

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A WEIGHTED DISCRETE UNIVERSALITY THEOREM FOR THE PERIODIC ZETA-FUNCTION

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Let $s = \sigma + it$ be a complex variable, and $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ be a periodic sequence of complex numbers. The periodic zeta-function $\zeta(s; \mathfrak{a})$ is defined, for $\sigma > 1$, by the Dirichlet series

$$\zeta(s; \mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s},$$

and is meromorphically continued to the whole complex plane.

In [1], a weighted universality theorem on the approximation of a wide class of analytic functions was obtained. Our report is devoted to a discrete version of the above theorem.

Suppose that the function $w(u)$ is such that

$$\lim_{N\to\infty} V_N = \lim_{N\to\infty} \sum_{k=1}^{N} w(k) = +\infty,$$

moreover, $w(u)$ has a continuous derivative $w'(u)$ for $u \geqslant 1$ such that

$$\int_1^N u|w'(u)|\mathrm{d}u \ll V_N.$$

Let I_A denote the indicator function of $A \subset \mathbb{R}$, $\mathcal{K}$ be the class of compact subsets of the strip $\left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$, and $H_0(K)$, $K \in \mathcal{K}$, be the class of continuous non-vanishing functions on K which are analytic in the interior of K. Then the following theorem is true.

THEOREM 1. *Suppose that the function $w(u)$ satisfies the above hypotheses, the sequence $\mathfrak{a}$ is multiplicative, and that α, $0 < \alpha < 1$, and $h > 0$ are fixed. Let $K \in \mathcal{K}$ and $f(s) \in H_0(K)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{N\to\infty} \frac{1}{V_N} \sum_{k=1}^{N} w(k) I_{\left\{k : \sup\limits_{s\in K} |\zeta(s+ik^{\alpha}h;\mathfrak{a})-f(s)|<\varepsilon\right\}}(k) > 0.$$

REFERENCES

[1] R. Macaitienė, M. Stoncelis and D. Šiaučiūnas. A weighted universality theorem for periodic zeta-functions. *Math. Modell. Analysis*, **22** (1):95 – 105, 2017.

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INVESTIGATION OF SPECTRUM CURVES FOR STURM–LIOUVILLE PROBLEM WITH TWO-POINT NONLOCAL BOUNDARY CONDITION

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Let us investigate the Sturm–Liouville problem

$$-u'' = \lambda u, \quad t \in (0,1), \tag{1}$$

with one classical (2) and the second two-point (3) nonlocal boundary condition

$$u(0) = 0, \tag{2}$$
$$u(1) = \gamma u'(\xi), \tag{3}$$

with parameters $\gamma \in \mathbb{R}$ and $\xi \in (0,1)$.

Characteristic function for Sturm–Liouville problem with one classical and other nonlocal two-point boundary conditions is analysed in the papers [3; 4]. In these papers investigated constant eigenvalue point, complex and real characteristic functions. The critical points for such type nonlocal boundary problems are less investigated, but they are important for numerically analysis of complex eigenvalues and spectrum curves in complex plane. In the papers [1; 2; 5] the similar problems critical points is investigated for nonlocal two-point or integral boundary conditions.

We analyze Sturm–Liouville problem and investigate how distribution of the critical points of spectrum of this problem depends on the parameters γ and ξ of the nonlocal boundary conditions. Many results are presented as a graphs of characteristic functions and spectrum curves.

REFERENCES

[1] S. Pečiulytė, A. Štikonas. Distribution of the Critical and other Points in Boundary Problems with Nonlocal Boundary Condition. *Lith. Mathem. J.*, **47** spec. no.:405–410, 2007.

[2] S. Pečiulytė, O. Štikonienė, A. Štikonas. Investigation of Negative Critical Points of the Characteristic Functions for Problems with Nonlocal Boundary Condition. *Nonlin. Anal.: Modell. and Contr.*, **13** (4):467–490, 2008.

[3] K. Skučaitė, S. Pečiulytė, A. Štikonas. Investigation of Complex Eigenvalues for Stationary Problems with two-point Nonlocal Boundary Conditions. *Mathem. and Mathemat. Modell.*, (5):24–32, 2009.

[4] A. Skučaitė and A. Štikonas. Spectrum Curves for SturmLiouville Problem with Integral Boundary Condition. *Math. Model. Anal.*, **20** (6):802–818, 2015. http://dx.doi.org/10.3846/13926292.2015.1116470

[5] A. Skučaitė and A. Štikonas. Zeroes and Poles of a Characteristic Function for Sturm Liouville Problem with Nonlocal Integral Condition. *Lith. Math. J. Proc.*, LSM, Ser. A, (56):95–100, 2015. http://www.mii.lt/LMR/A/2015/17.htm.

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PARALLEL ALGORITHMS FOR THE NUMERICAL SOLUTION OF PROBLEMS WITH FRACTIONAL POWERS OF ELLIPTIC OPERATORS

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We consider the numerical solution of problems defined by the fractional powers of elliptic operators [1]. In recent decades, popularity of fractional-order models is constantly increasing. Fractional-order derivatives and operators are successfully applied in various problems of physics, chemistry, biochemistry, biology, hydrology, image processing and finance. Fractional-order operators are able to describe anomalous behaviours of various materials, which are in between the ideal solids and Newtonian fluids, such as granular materials, colloids, polymers, emulsions, sediments, biological materials, multiphase fluids, and others. The behaviour of these materials often does not obey to the standard gradient laws, such as, Fick's law of diffusion, Fourier's law of heat conduction, Newton's law of viscosity, Darcy's law of a fluid flow through the porous medium. The fractional-order models appear to be more adequate than the standard models in the description of the long range interactions, memory and hereditary properties of different substances.

Let Ω be a bounded domain in $\mathbb{R}^n$, $n \geq 2$ with a boundary $\partial\Omega$. Given a function $f = f(\vec{x})$, we seek function $u = u(\vec{x})$ such that

$$L^\beta u = f, \quad \vec{x} \in \Omega \tag{1}$$

with some boundary conditions on $\partial\Omega$, fractional power $0 < \beta < 1$ and the elliptic operator:

$$Lu = -\sum_{j=1}^{n} \frac{\partial}{\partial x_j} \left(k(\vec{x}) \frac{\partial u}{\partial x_j} \right).$$

There are several definitions of fractional powers of elliptic operators. One of the most popular is definition through the spectral decomposition. Let us denote by $\{\phi_k\}$, $k = 1, 2, \ldots, N$ the orthonormal basis (for convenience, here we restrict to the case of finite number of modes typical for discrete approximations)

$$L\phi_k = \lambda_k \phi_k.$$

Then the fractional powers of the elliptic operator are defined by [2]

$$L^\beta u = \sum_{k=1}^{N} \lambda_k^\beta w_k \phi_k, \tag{2}$$

where $w_k = (u, \phi_k)$. The direct computational implementation of this definition is very expensive. It requires the computation of all eigenvectors and eigenvalues of large matrices. Such spectral numerical algorithm can be used for practical computations if the fractional power of Laplace operator is solved in a rectangular domain, when the basis functions are known in advance and FFT techniques can be applied.

In our work, we employ different transformations of non-local problem with fractional power of Laplacian operator into some local differential problems:

- elliptic problem with a degenerate or singular weight [2];
- pseudo-parabolic problem [3];
- integral representation using standard Laplacian operator [4];
- transformation using the best uniform rational approximation [5].

The advantage of this approach is that due to the common use of partial differential equations of such types their numerical solution methods are well developed. The software packages for their numerical solution (including parallel multigrid methods) are subject to a long-time development and permanent improvements.

We develop discrete schemes for the numerical solution of obtained local differential problems using the finite volume method. Three of these problems are formulated in the space of higher dimension. Thus, application of parallel computing technologies is required. Domain decomposition and master-slave methods are used for the construction of parallel algorithms. Open source parallel mutigrid libraries are employed for the numerical solution of obtained linear systems [6].

It is important to note that these transformations lead to very different properties of developed parallel algorithms. We investigate and compare the scalability and efficiency of these parallel algorithms. The issues of optimal data partitioning and load balancing are addressed.

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REFERENCES

[1] C. Pozrikidis. *The Fractional Laplacian*. Chapman and Hall/CRC, 2016.

[2] R. Nochetto, E. Otarola, and A. Salgado. A PDE approach to fractional diffusion in general domains: a priori error analysis. *Foundations of Computational Mathematics*, **15** (3):733–791, 2015.

[3] P. Vabishchevich. Numerical solving unsteady space-fractional problems with the square root of an elliptic operator. *Mathematical Modelling and Analysis*, **21** (2):220–238, 2016.

[4] A. Bonito and J. Pasciak. Numerical approximation of fractional powers of elliptic operators. *Mathematics of Computation*, **84** 2083–2110, 2015.

[5] S. Harizanov, R. Lazarov, P. Marinov, S. Margenov, and Y. Vutov. Optimal solvers for linear systems with fractional powers of sparse SPD matrices. *ArXiv e-prints*, (1612.04846): 1–25, 2016.

[6] R. Falgout, J. Jones, and U. Yang. The design and implementation of HYPRE, a library of parallel high performance preconditioners. In: *Numerical Solution of Partial Differential Equations on Parallel Computers*, Lecture Notes in Computational Science and Engineering, vol. 51, Springer Berlin Heidelberg, 2006, 267–294.

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INVESTIGATION OF A STATIONARY PROBLEM WITH TWO NONLOCAL CONDITIONS

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We investigate the second-order problem with two additional Nonlocal Conditions(NC):

$$\mathcal{L}u := -(p(t)u')' + q(t)u = f(t), \tag{1}$$
$$\langle L_0, u\rangle = f_1, \tag{2}$$
$$\langle L_1, u\rangle = f_2, \tag{3}$$

where $p(x) \geq p_0 > 0$, $p \in C^1[0,1]$, $q \in C[0,1]$, $\mathcal{L}\colon C^2[0,1] \to C[0,1]$, $L_i \in (C^2[0,1])^*$, $i = 0, 1$. Suppose, we know the fundamental system $\{u_0, u_1\}$ for the homogeneous differential equation. We define a determinant:

$$D(L_0, L_1)[u_0, u_1] := \begin{vmatrix} \langle L_0, u_0\rangle & \langle L_1, u_0\rangle \\ \langle L_0, u_1\rangle & \langle L_1, u_1\rangle \end{vmatrix}.$$

If $D(L_0, L_1)[u_0, u_1] \neq 0$ then an ordinary Green's function $G(t, s)$ and biorthogonal basis $\{v_0, v_1\}$ for $\{L_0, L_1\}$ exist [1].

Corresponding Sturm–Liouville problem is ([2])

$$\mathcal{L}u = \lambda u, \tag{4}$$
$$\langle L_0, u\rangle = 0, \tag{5}$$
$$\langle L_1, u\rangle = 0. \tag{6}$$

Equality $D(L_0, L_1)[u_0(\lambda), u_1(\lambda)] = 0$ describes eigenvalues for this Sturm–Liouville problem, where $\{u_0(\lambda), u_1(\lambda)\}$ a fundamental system for operator $\mathcal{L} - \lambda\mathrm{I}$.

We are analysed problems (1)–(3) and (4)–(6) for various $\mathcal{L}$, L_0, L_1 and generalized results for m-order problems.

REFERENCES

[1] S. Roman and A. Štikonas. Green's Functions for Stationary Problems with Nonlocal Boundary Conditions. *Lith. Math. J.*, **49** (2):190–202, 2009.

[2] A. Štikonas. Investigation of Characteristic Curve for Sturm–Liouville Problem with Nonlocal Boundary Conditions on Torus. *Math. Model. Anal.*, **16** (1):1–22, 2011.

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MODELLING OF NONLOCAL PROBLEM ON THE GRAPH FOR A FLOW IN A TUBE STRUCTURE

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An asymptotic analysis of the time dependent three dimensional Navier-Stokes equation in a thin tube structure [1; 2] leads to two types of equations for the pressure on the graph of the structure: one of them is a well-known Reynolds equation on the graph with Kirchhoff junction conditions at the nodes (it appears at the slow time scale) ; another equation is a new one proposed by G.Panasenko and K.Pileckas in [2; 3]. It corresponds to the fast time scale and couples a one-dimensional, nonlocal in time problem on the graph with a heat equation in the cross-section of the tubes. In the present talk, a numerical finite element scheme is proposed for this problem and its convergence is proved.

The method of asymptotic partial domain decomposition for thin tube structures (finite unions of thin cylinders) was developed in [1; 2]. Thin structures are some finite unions of thin rectangles (in $2D$ settings) or cylinders (in 3D settings) depending on small parameter that is, the ratio of the thickness of the rectangle (cylinder) to its length. The Navier–Stokes equations are considered in thin structures with the no-slip boundary condition at the lateral boundary and with the inflow and outflow conditions with the given velocity. To illustrate the theoretical results some computer simulations of flows in 2D tube systems are presented. Details of numerical approach and some technical aspects of asymptotic analysis will be discussed.

REFERENCES

[1] G. Panasenko and K. Pileckas. Asymptotic analysis of the non-steady Navier-Stokes equations in a tube structure.I. The case without boundary layer-in-time. *Nonlinear Analysis, Series A, Theory, Methods and Applications*, **122** :125–168, 2015. http://dx.doi.org/10.1016/j.na.2015.03.008

[2] G. Panasenko and K. Pileckas. Asymptotic analysis of the non-steady Navier-Stokes equations in a tube structure. II. General case. *Nonlinear Analysis, Series A, Theory, Methods and Applications*, **125**:582–607, 2015. http://dx.doi.org/10.1016/j.na.2015.05.018

[3] G. Panasenko and K. Pileckas. Flows in a tube structure: equation on the graph. *Journal of Mathematical Physics*, **55**:081505, 2014. http://dx.doi.org/10.1063/1.4891249

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A MODEL OF PYROLYSIS OF WOOD-STRAW MIXTURE [1]

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Biomass is a locally available energy resource that can contribute to reduction of greenhouse gas and pollutant emissions when applied for heat generation, e.g., for district heating. In order to optimize the management of local resources, biomass pellets are composed of various materials - wood, wheat straw, rape straw, peat etc. The generation of heat and the emissions strongly depend on the composition of the biomass as well as on other factors of the gasification and combustion process [1], [2].

When subjected to high temperature, the solid biomass releases volatile gases through the pyrolysis process of cellulose, hemicellulose and lignin. This study focuses on development of a mathematical model of the pyrolysis that is sensitive on variations in the composition of the wood-straw mixture and the type of straw and wood used.

The model will be applied in simulations of the combined gasification - combustion process (as in [3]) and for optimization of the composition of the biomass fuel.

REFERENCES

[1] Y. Shao, J. Wang, F. Preto, J. Zhu and C. Xu. Ash deposition in biomass combustion of co-firing for power/heat generation . *Energies*, **5** 5171–5189, 2012.

[2] M. Abricka, I. Barmina, R. Valdmanis, M. Zake and H. Kalis. Experimental and numerical studies on integrated gasification and combustion of biomass . *Chem. Eng. Trans.*, **50** 127–132, 2016.

[3] K. Kwiatkowski, P. Zuk, M. Dudynski and K. Bajer. Pyrolysis and gasification of single biomass particle - new openFoam solver . *J. Physics: Conf. series*, **530** 12–15, 2014.

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MATHEMATICAL MODELS AND ALGORITHMS FOR CREATION AND OPTIMIZATION OF BOLUS IN RADIOTHERAPY

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A radiation therapy (RT) is one of the most popular methods for the treatment of cancer. Approximately 45-55% of patients are treated using this method. In contemporary RT a tumor is defined very accurately applying modern computer tomography scanners and then the radiation beam can be directed exactly to the tumor region. The biggest challenge is that such a beam is damaging also human tissues along its path before and behind the tumor. A dose of radiation obtained by tissues behind the tumor can be reduced essentially by selecting appropriate intensity and energy of ionizing particles. This situation is particularly important when the tumor is positioned very close to other critical human organs. Despite big cost of hardware and software devices such a technology is already developed for ion radiation. At the same time for the electron radiation such a technology is still not developed, though it is well known that electron radiation would have much better properties in minimizing the damage of radiation to neighbor human organs. As a consequence patients are not always getting a treatment, which would be much more conservative for other critical organs during RT process [1]. The task to compute specific thickness and shape of bolus, adapted to details of the given patient and to select optimal multiple angles of treatment beams is challenging and very calculation-intensive.

In the report new VGTU- NCI project will be presented, which aim is creation, realization and verification of mathematical models and algorithms for radiation dose distribution in body tissues and for bolus geometry optimization. Application of simplified models makes possible to solve the global optimization problem to construct optimal thickness and shape of bolus, including the possibility to determine optimal angles of treatment beams. As a starting point we propose to use Fermi-Eyges-Hogstrom model [2].

REFERENCES

[1] Su S., Moran K., Robar J. L.. Design and production of 3D printed bolus for electron radiation therapy. *Journal of Applied Clinical Medical Physics*, **15** (4):194–211, 2014.

[2] Hogstrom, K. R., Almond, P. R.. Review of electron beam therapy physics. . *Physics in medicine and biology*, **51** (13): R455–R489, 2006.

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TRUNCATED SVD FOR COMPACT OPERATORS IN HILBERT SPACES AND ITS APPLICATION IN INVERSE PROBLEMS OF GEOPHYSICS

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Inverse problems in geophysics usually lead to non linear operator equation $B[m] = d$ with operator $B : M \to D$ mapping a model space M to a data space D. We use modified Newton technique to resolve this equation:

$$m_{k+1} = m_k + \alpha_k \left(DB[m_k]\right)_r^{\dagger} < d - B[m_k] > \tag{1}$$

Here DB is Frechet derivative of the nonlinear map B. Usually this operator is compact and, hence, has no bounded inverse. To organize iteration process we use r-pseudo inverse constructed on the base of truncated Singular Value Decomposition (SVD) of compact operator DB:

$$\left(DB[m_k]\right)_r^{\dagger} < d >= \sum_{n=1}^{r} \frac{(d, \psi_n)_D}{s_n} \phi_n \tag{2}$$

The following theorem describes regularity of r-pseudo inverse operator:

THEOREM 1. *Consider two equations $Ax = y$, $\widetilde{A}\widetilde{x} = \widetilde{y}$ with compact operators and right hand sides*

$$\|A - \widetilde{A}\| \le \delta \|A\|, \quad \|y - \widetilde{y}\| \le \epsilon \|y\|.$$

Let us suppose there is a gap in the singular spectrum of A, $2d_r\delta < 1$, $\quad d_r = \|A\| \left(s_r - s_{r+1}\right)^{-1}$. Then r-solutions $x_r =$ and $\widetilde{x}^r$ of these equations exist and the following estimation is hold

$$\frac{\|x^r - \widetilde{x}^r\|}{\|x^r\|} \le \frac{s_1}{s_r} \frac{(\rho + \tau)(1 + \beta\delta)}{1 - \mu_r(A)\rho}.$$

in terms of the following parameters

$$\theta = \theta_r(A, g) = \frac{\|Af_{[r]} - g\|}{s_r \|f_{[r]}\|}, \qquad \mu_r(A) = \frac{s_1}{s_r}, \beta = \frac{\sqrt{2}d_r}{\sqrt{1 - d_r\delta}\sqrt{1 - d_r\delta\sqrt{1 - 2d_r\delta}}}, \qquad \rho = \delta(1 + 2\beta),$$

$$\tau = \sqrt{1 + \theta^2} \left(\epsilon + \frac{d_r\delta}{1 - d_r\delta} + \frac{\sqrt{2}d_r\delta}{\sqrt{1 - d_r\delta}\sqrt{1 - d_r\delta\sqrt{1 - 2d_r\delta}}} \right)$$

under constraint

$$\mu_r(A)\rho < 1.$$

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CONSTRUCTION OF SOLITARY SOLUTIONS TO RICCATI EQUATIONS WITH MULTIPLICATIVE AND DIFFUSIVE COUPLING USING OPERATOR TECHNIQUES

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With the development of computer algebra software over the recent years, a number of mathematical techniques for the construction of analytical solutions to nonlinear ordinary differential equations (ODEs) and partial differential equations (PDEs) have been introduced. The generalized differential operator method is a powerful mathematical tool that can be used to construct closed-form solitary solutions to differential equations in research fields ranging from astrophysics to mathematical biology [1; 2].

In this presentation, we consider a system of Riccati equations coupled with both multiplicative and diffusive terms:

$$\begin{aligned} &x_t' = a_0 + a_1 x + a_2 x^2 + a_3 xy + a_4 y; \\ &y_t' = b_0 + b_1 y + b_2 y^2 + b_3 xy + b_4 x; \\ &x\Big|_{t=t_0} = x_0; \quad y\Big|_{t=t_0} = y_0, \end{aligned} \tag{1}$$

where $a_k, b_k \in \mathbb{R}, k = 0, \ldots, 4$. System (1) is used in mathematical biology for the modeling of population dynamics with the Allee effect [3] and hepatitis C virus evolution [4].

Using the generalized differential operator technique it is shown that system (1) admits both kink and bright/dark solitary solutions for all initial conditions t_0, x_0, y_0 when certain constraints on the parameters $a_k, b_k, k = 0, \ldots, 4$ hold true. The derivation of these constraints, construction of solitary solutions as well as the dynamics of (1) are addressed in this presentation. It is also shown that (1) cannot admit solitary solutions of higher order than bright/dark solitons.

REFERENCES

[1] R. Marcinkevicius, Z. Navickas, M. Ragulskis and T. Telksnys. Solitary solutions to a relativistic two-body problem. *Astrophysics and Space Science*, **361** 201-209, 2016.

[2] Z. Navickas, R. Marcinkevicius, T. Telksnys and M. Ragulskis. Existence of second order solitary solutions to Riccati differential equations coupled with a multiplicative term. *IMA Journal of Applied Mathematics*, **81** 1163-1190, 2016.

[3] F. Courchamp, L. Berec and J. Gascoigne. *Allee effects in ecology and conservation.* Oxford University Press, Oxford, 2008.

[4] T. C. Reluga, H. Dahari and A. S. Perelson. Analysis of hepatitis c virus infection models with hepatocyte homeostasis. *SIAM Journal on Applied Mathematics*, **69** 999-1023, 2009.

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NONLINEAR THERMODIFFUSION IN GASES AT MODERATE TEMPERATURES

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In earliest papers A.J. Janaviius proposed the nonlinear diffusion equation [1] with the diffusion coefficient directly proportional to the concentration of impurities. This equation provides a more exact description of the profiles of impurities in Si crystals. The heat transfer in gases carry a greater average kinetic energy of gas molecules from hot regions to the coldest ones with a finite velocity by random Brownian motions. In this case the heat transfer in gases can be described by using nonlinear thermodiffusion equation for heat transfer when thermodiffusion coefficients are directly proportional to temperature T. The obtained approximate analytical solutions are successfully applied for defining temperature profiles and heat transfer coefficients in gases. The obtained solutions can be applied for practical applications. It has been concluding that heat spreading in gases depends on temperature differences and pressure in gases. The equation of thermal conductivity of gases for one-dimensional case with the nonlinear thermodiffusion coefficient $D_n(T) = D_{en}T$ proportional to the temperature is written similarity as for diffusion of impurities [1], [2]

$$\frac{\partial T}{\partial t} = -div(-D_n(T)grad(T)), j_p = -D_n(T)\frac{\partial T}{\partial x}, \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\Big(D_n(T)\frac{\partial T}{\partial x}\Big) \tag{1}$$

Here D_{en} - constant of the proporcionality for nonlinear thermodiffusion coefficient. The maximum value of similarity variable ξ_0, defining a maximum of heat penetration depths x_0 , can be expressed approximately by difference ΔT of source T_s and environment T_e temperatures

$$x_0 = \xi_0\sqrt(D_e t), \xi_0 = -\frac{1}{a_1}(T_s - T_e)/T_e, \xi_0 = \frac{x_0}{\sqrt(D_e T_e t)} = \frac{x_0}{\sqrt(D_e t)} \tag{2}$$

The thermodiffusion coefficients D_e depend on T_e and frequencies of molecules collisions in the frontier region. The results show that heat penetration depths and ξ_0 [1] are approximately proportional to $\frac{\Delta T}{T_e}$ values.

REFERENCES

[1] A.J.Janavičius. Method for solving the nonlinear diffusion equation. *Physics Letters A*, **224** 159-162, 1997.

[2] A.J.Janavičius, S.Turskienė. Nonlinear thermal conductivity in gases. *Proc. of the Lithuanian Mathematical SocietyJournal*, **57** 21-28, 2016.

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NUMERICAL SOLUTION OF TIME-DEPENDENT PROBLEMS WITH FRACTIONAL POWER ELLIPTIC OPERATOR

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An unsteady problem is considered for a space-fractional equation in a bounded domain. A first-order evolutionary equation involves a fractional power of an elliptic operator of second order. Finite element approximation in space is employed. To construct approximation in time, standard two-level schemes are used. The approximate solution at a new time-level is obtained as a solution of a discrete problem with the fractional power of the elliptic operator. A Pade-type approximation is constructed on the basis of special quadrature formulas for an integral representation of the fractional power elliptic operator using explicit schemes. A similar approach is applied in the numerical implementation of implicit schemes. The results of numerical experiments are presented for a model two-dimensional problem.

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ON PSEUDO-DIFFERENTIAL EQUATIONS IN DISCRETE SPACES

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We introduce a class of symbols $A(x,\xi)$ which are defined on $\mathbb{R}^m \times \mathbb{R}^m$ and are periodic on a variable ξ with the basic cube of periods $\mathbb{T}^m = [-\pi,\pi]^m$. This defines a discrete pseudo-differential operator by the formula

$$(Au_d)(\tilde{x}) = \int\limits_{\mathbb{T}^m} A(\tilde{x},\xi)\tilde{u}_d(\xi)e^{i\tilde{x}\cdot\xi}d\xi,$$

where u_d is a function of discrete variable $\tilde{x} \in \mathbb{Z}^m$, $\tilde{u}_d$ is its Fourier transform.

Our main goal is studying solvability for equations with such operators of the following type

$$(A_d u_d)(\tilde{x}) = v_d(\tilde{x}), \quad \tilde{x} \in D_d,$$

where $D_d = D \cap \mathbb{Z}^m$, D is a domain in $\mathbb{R}^m$.

We introduce corresponding discrete Sobolev–Slobodetskii spaces $H^s(D_d)$, and according to a local principle first we are interested in studying the invertibility of model operators with symbols non-depending on $\tilde{x}$ ($A(\tilde{x},\xi) \equiv A(\xi)$) in canonical discrete domains ($D = \mathbb{R}^m, \mathbb{R}^m_+ = \{x \in \mathbb{R}^m : x = (x', x_m), x_m > 0\}, C^a_+ = \{x \in \mathbb{R}^m : x_m > a|x'|, a > 0\}$).

First the first canonical case $\mathbb{R}^m$ one needs the discrete Fourier transform and an appropriate norm in discrete H^s-space. For the second one $\mathbb{R}^m_+$ it is not enough and one adds the theory of periodic Riemann boundary value problem [3]. Finally the case C^a_+ requires a certain multidimensional variant of periodic Riemann problem [4].

One can note that there are a lot of correspondences between pseudo-differential theory of boundary value problems, the theory of multidimensional difference equations and discrete equations. Some considerations for certain simplest classes of difference equations were done in [1; 2] for a discrete half-space case, and more general classes and more complicated conical case was initiated in [4].

REFERENCES

[1] A. Vasilyev, V. Vasilyev. Difference equations in a multidimensional space. *Math. Model. Anal.*, **21** 3: 336–349, 2016.

[2] A. Vasilyev, V. Vasilyev. Difference equations and boundary value problems. In: *Springer Proc. Math. & Stat.* **164**, Differential and Difference Equations and Applications, S. Pinelas et al. (Eds.), Birkhäuser, Basel, 2016, 421–432.

[3] A. Vasilev, V. Vasilev. Periodic Riemann problem and discrete convolution equations. *Differential Equations*, **51** 5: 652–660, 2015.

[4] V. Vasilyev. Discrete equations and periodic wave factorization. In: *Proceedings of International Conference on Analysis and Applied Mathematics, Almaty, Kazakhstan, 2016*, AIP Conference Proceedings **1759**, 2016, 5 pp..

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MULTIPLICITY RESULTS FOR PERIODIC BVP TO THE SECOND ORDER ASYMPTOTICALLY LINEAR SYSTEM

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We consider the second order asymptotically linear system

$$x_i'' = \mu_i x_i + \phi'\left(\frac{1}{2}\sum_{i=1}^{n}\eta_i x_i^2\right)\eta_i x_i, \qquad i=\overline{1,n}, \tag{1}$$

where all η_i are positive (or negative) and regulatory function ϕ satisfies the following properties:
(P$_1$) $\phi \in C^2(I,\mathbb{R})$, where $I=[0,+\infty)$ or $I=(-\infty;0]$;
(P$_2$) $\phi(0)=0$;
(P$_3$) $\phi'(0)=1$;
(P$_4$) $\lim\limits_{s\to\infty}\phi'(s)=0$;
(P$_5$) in each finite interval $[0;\,a]$ (or $[-a;\,0]$) $\phi'(s)$ has a finite number of extrema.

Theorem 1. *If the following inequality holds*

$$\prod_{i=1}^{n}\mu_i(\mu_i+\eta_i)<0 \tag{2}$$

then there exist non-zero singular points of the system (1).

The system (1) is considered subject to the periodic boundary conditions

$$x_i(0)=x_i(T), \qquad x_i'(0)=x_i'(T) \qquad i=\overline{1,n}. \tag{3}$$

We investigate for which $T>0$ there exist "small" and "large" periodic solutions of the problem (1), (3).

REFERENCES

[1] V.I. Arnold. *Mathematical Methods of Classical Mechanics*. Springer, 1989.

[2] S. Atslega, F. Sadyrbaev. Period annuli and multiple solutions for two-point BVP. *Tatra Mountains Math. Publ.*, (43):11–23, 2009.

[3] M. Berti. *Nonlinear Oscillations of Hamiltonians PDEs*. Birkhäuser, Basel, 2007.

[4] P.P. Zabrejko. Rotation of vector fields: definition, basic properties, and calculation. In: *Progress in Nonlinear Differential Equations and Their Applications, v. 27*, (Eds: M. Matzeu, A. Vignoli), Birkhäuser, Boston-Basel-Berlin, 1997, 445–601.

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EXTRACTION OF VECTOR FEATURES BY NUMERICAL QUANTIZATION WITH VARIABLE THRESHOLD

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The numerical problem of extraction of vector features is a well-known in computer vision or medical engineering [1] and used to description of different objects, especially in original Big Data matrix $\mathbf{A}$. First, we have problem of finding important subset $\mathbf{f}_i$ to identify essential properties of $\mathbf{A}$. The paper proposes new method of searching a small subset of features. We use variable threshold $\Delta_L = L_{j+1} - L_j$ with a floating value Δ_L for each vector $\mathbf{f}_i$ to initial division of $\mathbf{A}$.

The approach is concerned with the identification of the most important biomarkers and their application in cancer recognition [2], [3]. Search features are often difficult to identify because of data complexity and their redundancy ($\mathbf{A}_{mxn}$, $m \ll n$ and $n < \infty$). First, we estimate the non-uniformity density $\delta_{L_{num}}(f_i)$ (1) for deep exploration of structure of $\mathbf{f}_i$ separately

$$\delta_{L_{num}}(f_i) = \sum_{j=1}^{q_{num}^{I_c}} (k_L \frac{N_{\Delta_L}}{N_c}) \tag{1}$$

end find c class coefficient $\Phi^c(f_i)$

$$\Phi^c(f_i) = \sum_{j=1}^{v} \delta_{L_{num}}(f_i), \tag{2}$$

for calculate indicator of attribute $\mathbf{f}_i$:

$$\Delta\Phi^c(f_i) = |\Phi^{c=1}(f_i) - \Phi^{c=2}(f_i)|. \tag{3}$$

To separate and find hidden attributes $\mathbf{f}_i$ we use a special algorithm QTFV of adjustable width for the quantum intervals for this feature. Finally, this procedure generates a small subset $C(\mathbf{E}_r)$ of representative and essential attributes. The content of the subset $C(\mathbf{E}_r)$ may be varied, depending on size of the defined target.

REFERENCES

[1] I. Guyon, A. Elisseeff. An introduction to variable and feature selection. *J. Machine Learning Research*, (3):1158-1182, 2003.

[2] A. Wilinski, S. Osowski. Gene selection for cancer classification. *COMPEL - The international journal for computation and mathematics in electrical and electronic engineering*, **28** (1):231–241, 2009.

[3] A. Wilinski, S. Osowski. Ensemble of data mining methods for gene ranking. *Bulletin of the Polish Academy of Sciences*, **60** (3):461–471, 2012.

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MECHANICS-MATHEMATICAL MODEL OF CANCER

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Cancer is cause of specific changes of cells in their biomechanical properties along malignant transformation [1]. Increased deformability cells identifies higher malignancy and metastatic potential in many different cancers. Mechanics-mathematical model has been derived to define changes of biomechanical properties of cancer cells. Nowadays nanoindentation and atomic force microscopy (AFM) are widely used to research biomechanical properties of cancer cells, especially cell elasticity and deformability on micron and submicron scales [2]. One of majority problem AFM using technologies is choice/design of mathematical model to describe the cells' changes and to take into account the structure of real heterogeneous medium, actions of different forces, adhesive and etc. [3,4] In present work using of linear viscoelastic equation with exponential law of creep, the dependence in which stress is proportional to fractional order derivative of deformation has been obtained in following form

$$\sigma(t) = \eta({}_0D_t^{\alpha}\varepsilon(t)), \eta = \frac{1}{A\Gamma(1+\alpha)} \tag{1}$$

where ${}_0D_t^{\alpha}\varepsilon(t)$ is Riemann-Liouville fractional derivative [3].

Modification of Cricks algorithm and technique of determination of contact point have been fulfilled. Fractional order model (1) has been used to AFM-probe with different cross-section shape and bases. Effective elastic modulus has been defined on following obtained formulas [4]:

$$E = \frac{3P(1-\mu^2)}{4\sqrt{R}({}_0D_t^{\alpha}h(t))^{1,5}} \tag{2}$$

$$E = \frac{P(1-\mu^2)}{2(a\ {}_0D_t^{\alpha}h(t) - \frac{a^2}{2\operatorname{tg}\theta}\arccos\frac{b}{a} - \frac{a^3}{3R} + \sqrt{a^2-b^2}(\frac{b}{2\operatorname{tg}\theta} + \frac{a^2-b^2}{3R})} \tag{3}$$

To solve the problem of the interaction of an indenter to erythrocytes cells of cancer patients there was carried out a numerical model for the real data. Comparative analysis of numerical, analytical and experimental results has been presented in this work.

REFERENCES

[1] S.Suresh Biomechanics and biophysics of cancer cells. Acta Biomater, 3(4): 413438, 2007

[2] M. Zhuravkov, L. Drozd, N. Romanova, A. Krupoderov Mechanical-mathematical modelling of biological tissue behavior. Springer Proc. in Mathem. & Statistics, 116: 153181, 2015.

[3] M. Zhuravkov, N. Romanova Review of methods and approaches for mechanical problem solutions based on fractional calculus. Mathematics and Mechanics of Solids, 21: 595620, 2016.

[4] M. Zhuravkov, N. Romanova Determination of physical and mechanical properties of biomaterials on base of the nanoindentation technologies and fractional order models. Russian Journal of Biomechanics, 20: 522, 2016.

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IMPROVEMENT OF DIRECT-TYPE ALGORITHMS

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We consider global optimization algorithms derived from the well known DIRECT (DIviding RECTangles) algorithm [1]. This algorithm involves evaluation of the objective function at the centers of hyper-rectangles and division of potentially optimal ones. Various modifications of the DIRECT algorithm have been proposed including different partitions, various sampling strategies, and balancing of global and local search.

Although most of DIRECT-type algorithms use hyper-rectangular partitions, simplicial partitions (DISIMPL algorithm) [6; 7] have several advantages [8]. Central sampling of the objective function can be changed to diagonal approach sampling at the endpoints of diagonal [2; 9; 10]. Trisection is usually used to enable reuse of the objective function values at the center or endpoints of diagonals in descendant subregions. Bisection can ensure better shapes of hyper-rectangles with smaller variation of sizes in different dimensions than trisection which produces sizes differing by three times, but a special sampling strategy is necessary to enable reuse of sample points [4]. DIRECT-type algorithms often spend an excessive number of function evaluations exploring suboptimal local minima and delaying discovery of the global minimum. A significant speed-up may be achieved for the DIRECT-type algorithms with two-phase technique [5; 9]. Another modification significantly speeding-up the algorithm is reducing the set of potentially optimal hyper-rectangles [3]. In this talk we overview such modifications and discuss improvements.

REFERENCES

[1] D. R. Jones, C. D. Perttunen and B. E. Stuckman. Lipschitzian optimization without the Lipschitz constant. *Journal of Optimization Theory and Application*, **79** (1):157–181, 1993.

[2] D. E. Kvasov, C. Pizzuti and Ya. D. Sergeyev. Local tuning and partition strategies for diagonal GO methods. *Numerische Mathematik*, **94** (1):93–106, 2003.

[3] J. Mockus, R. Paulavičius, D. Rusakevičius, D. Šešok, J. Žilinskas. Application of reduced-set Pareto-Lipschitzian optimization to truss optimization. *Journal of Global Optimization*, **67** (1):425–450, 2017.

[4] R. Paulavičius, L. Chiter and J. Žilinskas. Global optimization based on bisection of rectangles, function values at diagonals, and a set of Lipschitz constants. *Journal of Global Optimization*, in press, 2016.doi:10.1007/s10898-016-0485-6

[5] R. Paulavičius, Ya. D. Sergeyev, D. E. Kvasov and J. Žilinskas. Globally-biased DISIMPL algorithm for expensive global optimization. *Journal of Global Optimization*, **59** (2-3):545–567, 2014.

[6] R. Paulavičius and J. Žilinskas. Simplicial Lipschitz optimization without the Lipschitz constant. *Journal of Global Optimization*, **59** (1):23–40, 2013.

[7] R. Paulavičius and J. Žilinskas. *Simplicial Global Optimization*. Springer, New York, 2014.

[8] R. Paulavičius and J. Žilinskas. Advantages of simplicial partitioning for Lipschitz optimization problems with linear constraints. *Optimization Letters*, **10** (2):237–246, 2016.

[9] Ya. D. Sergeyev and D. E. Kvasov. Global search based on efficient diagonal partitions and a set of Lipschitz constants. *SIAM Journal on Optimization*, **16** (3):910–937, 2006.

[10] Ya. D. Sergeyev and D. E. Kvasov. *Diagonal Global Optimization Methods*. FizMatLit, Moscow, 2008. In Russian

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FAST FFT-BASED SOLVERS FOR HIGHER ORDER FEM ON RECTANGULAR PARALLELEPIPEDS FOR PDE

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We present direct fast algorithms to implement nth order ($n \geq 2$) finite element method (FEM) on rectangular parallelepipeds [1] for solving a N-dimensional generalized Poisson equation ($N \geq 2$) with the Dirichlet boundary condition. The algorithms are based on the well-known Fourier approaches. The key new points are the fast direct and inverse algorithms for expansion in eigenvectors of the 1D eigenvalue problems for the high order FEM utilizing several versions of the discrete fast Fourier transform (FFT). This solves the old known problem, see [2, p. 271], and makes the full algorithms logarithmically optimal with respect to the number of elements as in the case of the bilinear elements ($n = 1$) or standard finite-difference schemes. The algorithms are fast in practice (faster than the theoretical expectations) and demonstrate only a mild growth in n starting from the standard case $n = 1$. For example, in the 9th order case, the 2D FEM system for 2^{20} elements containing almost $85 \cdot 10^6$ unknowns and the 3D FEM system for 2^{18} elements containing more than $190 \cdot 10^6$ unknowns are solved respectively in less than 2 and 15 min on an ordinary laptop using Matlab R2016a code. See [3] for more details. The algorithms are also highly parallelizable.

They can further serve for a variety of applications including general 2nd order elliptic equations (as preconditioners), for the N-dimensional heat, wave or time-dependent Schrödinger PDEs, etc. They can be applied for some non-rectangular domains, in particular, by using meshes topologically equivalent to rectangular ones. Other standard boundary conditions can be covered as well, see a brief description in [4]. Moreover, the Fourier structure of algorithms is especially valuable for solving some wave and nuclear physics problems, in particular, in 2D and 3D unbounded domains involving transparent boundary conditions, e.g., see [2; 5; 6], whence our own interest arose.

Results of numerical experiments for $N = 2$ and 3 are presented in detail; all of them include the standard case $n = 1$ for comparison.

REFERENCES

[1] P.G. Ciarlet. *Finite element method for elliptic problems*. SIAM, Philadelphia, 2002.

[2] B. Bialecki, G. Fairweather and A. Karageorghis. Matrix decomposition algorithms for elliptic boundary value problems: a survey. *Numer. Algor.*, **56** :253–295, 2011.

[3] A. Zlotnik and I. Zlotnik. On fast Fourier solvers for the tensor product high-order FEM for a generalized Poisson equation. *https://arxiv.org/abs/1701.03967*, 2017, 1–20.

[4] A.A. Zlotnik and I.A. Zlotnik. A fast direct algorithm for implementing a high-order finite element method on rectangles as applied to boundary value problems for the Poisson equation. *Doklady Mathematics*, **95** (2), 2017.

[5] B. Ducomet, A. Zlotnik and I. Zlotnik. The splitting in potential Crank-Nicolson scheme with discrete transparent boundary conditions for the Schrödinger equation on a semi-infinite strip. *ESAIM: Math. Model. Numer. Anal.*, **48** (6):1681–1699, 2014.

[6] A. Zlotnik and I. Zlotnik. Finite element method with discrete transparent boundary conditions for the time-dependent 1D Schrödinger equation. *Kinetic Relat. Models*, **5** (3):639–667, 2012.

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